

Konno-Yamazaki Portfolio Optimization Model and an Application to Istanbul Stock Exchange

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Received: May 11, 2009

Abstract. The portfolio selection problem deals with how to form a satisfying portfolio. It is difficult to decide which assets should be selected because of the uncertainty on their returns. On the other hand, the increased volatility of financial markets during the last decade has induced researchers, practitioners and regulators to design and develop more sophisticated risk management tools. Value at Risk (VaR) has become the standard measure that financial analysts use to quantify market risk. VaR is defined as the maximum potential loss in value of a portfolio due to adverse market movements, for a given probability. In this work we considered Konno-Yamazaki model to optimize our portfolio and we compare this optimal portfolio with other possible portfolios with respect to VaR values.

Key words: Portfolio optimization, Konno-Yamazaki model, Value at Risk.

2000 Mathematics Subject Classification: 90C05, 91B28, 62G32.

1. Introduction

The selection of assets or equities is not just a problem of finding attractive investments. Designing the correct portfolio of assets cannot be done by human intuition alone and requires modern, powerful and reliable mathematical programs called optimizers. Any investor would like to have the highest return possible from an investment. However, this has to be counterbalanced by the amount of risk the investor is able or desires to take. The expected return and the risk measured by the variance (or the standard deviation, which is the square-root of the variance) are the two main characteristics of a portfolio. Unfortunately, equities with high returns usually correlate with high risk.

The mathematical problem of portfolio optimization was initiated by Professor Harry Markowitz in the fifties and he was rewarded with a Nobel Prize in

Economics in 1990. Later, modification of this portfolio optimization such as Konno-Yamazaki model is used for this purpose.

On the other hand, while choosing the optimal portfolio one must think about the risk aspect of the events. Measuring risk is the other problem which investments must be faced with. Value at risk theory has been designed for this reason.

In financial mathematics and financial risk management, Value at Risk (VaR) is a widely used measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading in the portfolio) is the given probability level. There are three basic approaches that are used to compute Value at Risk, though there are numerous variations within each approach. The measure can be computed analytically by making assumptions about return distributions for market risks, and by using the variances in and covariances across these risks. Variance-Covariance method, historical simulation method and Monte Carlo Simulation method are widely used to measure Value at risk.

2. Konno-Yamazaki Optimization Model

According to Konno and Yamazaki MAD portfolio optimization model's advantages over the Markowitz's model are: this model does not use the covariance matrix and so it does not need it to be calculated, solving this linear model is much easier than solving a quadratic model, the maximum number of assets that are invested in is $2T + 2$ (if $u_j = \infty$) where Markowitz's model may contain as many as n assets and T can be used as a control variable to restrict the number over the assets. Konno and Yamazaki (1991) introduced risk function (mean absolute deviation-MAD) instead of the risk (variance) function where R_j is a random variable representing the rate of return per period of the asset j . They proved that these two measures are the same if $(R_1 \dots R_n)$ are multivariate normally distributed. So the Konno-Yamazaki MAD portfolio optimization model becomes as follows; [4]

$$\text{Min}Z = \sum_{i=1}^T y_i/T$$

subject to

$$\begin{aligned}
& y_t + \sum_{j=1}^n a_{ij}x_j \geq 0, \quad t = 1, 2, \dots, T \\
& y_t - \sum_{j=1}^n a_{ij}x_j \geq 0, \quad t = 1, 2, \dots, T \\
(1) \quad & \sum_{j=1}^n r_jx_j \geq \rho M_0 \\
& \sum_{j=1}^n x_j = M_0 \\
& 0 \leq x_j \leq u_j, \quad j = 1, 2, \dots, n \\
& y_t \geq 0
\end{aligned}$$

The aim of the model is minimizing risk of the portfolio, and the variables in this model is defined as,

$$(2) \quad a_{ij} = r_{jt} - r_j \quad j = 1, 2, \dots, n, \quad t = 1, 2, \dots, T$$

T =number of period
 t = t^{th} period
 ρ =expected income
 r_j =expected return of j^{th} asset.
 r_{jt} =real return of j^{th} asset at t^{th} period
 x_j =share of each assets.
 M_0 =total investment
 y_t =assistant variable

3. Value at Risk

The most prominent of these risks in trading is market risk, since it reflects the potential economic loss caused by the decrease in the market value of a portfolio. Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk. It is defined as the maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain horizon. In simpler words, it is a number that indicates how much a financial institution can lose with probability θ over a given time horizon. The great popularity that this instrument has achieved among financial practitioners is essentially due to its conceptual simplicity: VaR reduces the (market) risk associated with any portfolio to just one number, that is the loss associated with a given probability. VaR measures can have many applications, such as in risk management, to evaluate the performance of risk takers and for regulatory requirements. In particular, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements imposes to financial institutions such as banks and investment firms to meet capital requirements based on VaR estimates. Providing accurate estimates is of crucial importance. If the underlying risk is

not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability or the financial stability of the institutions. [1]

While VaR is a very easy and intuitive concept, its measurement is a very challenging statistical problem. Although the existing models for calculating VaR employ different methodologies, they all follow a common general structure, which can be summarised in three points: 1) Mark-to-market the portfolio, 2) Estimate the distribution of portfolio returns, 3) Compute the VaR of the portfolio. The main differences among VaR methods are related to point 2, that is the way they address the problem of how to estimate the possible changes in the value of the portfolio. [5] CAViaR models skip the estimation of the distribution issue, as they allow computing directly the quantile of the distribution. We will classify the existing models into three broad categories:

1. Parametric (RiskMetrics and GARCH)
2. Nonparametric (Historical Simulation and the Hybrid model)
3. Semiparametric (Extreme Value Theory, CAViaR and quasi-maximum likelihood GARCH)

4. Application

In application, Istanbul Stock Market real datas are used to get optimal portfolio with Konno-Yamazaki optimization model and to calculate VaR values of the optimal portfolio. To solve the linear program for portfolio optimization WIN-QSB programming are used. The solution of portfolio optimization is shown in Table 1. 13 different assets closing prices between 01.01.2006 and 01.01.2009 are used. Konno-Yamazaki portfolio optimization model is calculated as,

$$\min Z = \left(\frac{1}{36}Y_1 + \frac{1}{36}Y_2 + \frac{1}{36}Y_3 + \dots + \frac{1}{36}Y_{36} \right)$$

subject to

$$\begin{aligned} & Y_1 + 2.632ULKER + 6.774VESTEL + 2.066TUKAS - 1.922MUTLU + \dots + 7.810VKFBNK \\ & Y_2 + 7.32ULKER + 3.379VESTEL + 7.312TUKAS + 3.816MUTLU + \dots + 0.443VKFBNK \\ & \vdots \\ & Y_{36} - 14.30ULKER - 11.86VESTEL - 10.92TUKAS - 11.06MUTLU + \dots - 26.93VKFBNK \\ & Y_T \geq 0, \quad T = 1, 2, 3, \dots, 36 \\ & X_i \geq 0, \quad i = 1, 2, \dots, 13 \end{aligned}$$

Decision Variables	Assets	Shares (%)
X1	ULKER	0,053
X2	VESTEL	0,038
X3	TUKAS	0,000
X4	MUTLU AKU	0,000
X5	MIGROS	0,498
X6	TOFAS	0,000
X7	TURKCELL	0,091
X8	YAPIKREDI	0,115
X9	GARANTI	0,000
X10	SEKERBANK	0,000
X11	FORTIS	0,205
X12	AKBANK	0,000
X13	VAKIFBANK	0,000

Table 1. Solution of the portfolio optimization model

According to the table 1 the investors should invest their money to %0.53 Ulker, %0.38 Vestel, %49.8 Migros, %9 Turkcell, %11.5 YapıKredi and %20.5 Fortis.

Then, with optimal portfolio VaR value are calculated. VaR value of the optimal portfolio with a given probability %95 with ten day period is %0.089, on the other hand with a given probability %95 and one-month period VaR of the optimal portfolio is %0.13. VaR is calculated with variance and covariance method.

5. Conclusion

For most investors, the risk they take when they buy a stock is that the return will be lower than expected. In other words, it is the deviation from the average return. Each stock has its own standard deviation from the mean, which Markowitz calls "risk".

The risk in a portfolio of diverse individual stocks will be less than the risk inherent in holding any single one of the individual stocks (provided the risks of the various stocks are not directly related). Consider a portfolio that holds two risky stocks: one that pays off when it rains and another that pays off when it doesn't rain. A portfolio that contains both assets will always pay off, regardless of whether it rains or shines. Adding one risky asset to another can reduce the overall risk of an all-weather portfolio.

In this work we considered Konno-Yamazaki model to optimize the portfolio of Istanbul Stock Exchange real data between 01.01.2006 and 01.01.2009. After

obtaining optimal portfolio we calculated VaR value of the optimal portfolio with a given weighted matrix and we concluded that Konno-Yamazaki portfolio optimization model gives the minimum risk value comparing with other possible weighted portfolios.

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