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The Maple Program Procedures at Solutıon of Differantial Equations with Taylor Collocation Method

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Summary. Numerical methods which are based on algorithm and given solutions fastly, are come into prominence for solution of differential equations are encountered in applied mathematics and some of enginering problems, don't have analytical solutions or have so difficult and time-consuming solutions. One of these methods is Taylor Collocation method. To solve complex situations are encountered and to acquire solutions of the other engineering problems are possible with the mentioned method.
In this study we show that are given definition of Taylor Collocation Method and calculations of this method, some of differential equations can solve via Maple computer programme

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1. Taylor Collocation Method

The Taylor method is developed to find an approximate solution of high-order linear differential-difference equations, integro differential equations with variable coefficients under the mixed conditions [2]. The solution is obtained in terms of Taylor polynomials. Firstly,this method is based on taking the truncated Taylor series of the function in equations and then substituting their matrix forms in the given equation. Hence, the result of matrix equation can be solved and the unknown Taylor coefficients can be found approximately [1,3,4].

m th-order linear differential equation with variable coefficients

$$\sum_{k=0}^{m} P_k(x) y^{(k)}(x) = f(x),\ a \leq x \leq b \tag{1}$$

with the mixed conditions

$$\sum_{j=0}^{m-1} [a_{ij} y^{(j)}(a) + b_{ij} y^{(j)}(b) + c_{ij} y^{(j)}(c)] = \lambda_i, i = 0, 1, \ldots, m-1;\ a \leq c \leq b \tag{2}$$

then we can write the equation (1)

$$\begin{aligned} P_m(x) y^{(m)}(x) + P_{m-1}(x) y^{(m-1)}(x) + \cdots + P_1(x) y^{(1)}(x) + P_0(x) y = \\ = f(x), a \leq x, c \leq b \end{aligned} \tag{3}$$

and the approximate solution is expressed in the truncated Taylor series,

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!} (x-c)^n;\ a \leq x, c \leq b,\ N \geq m \tag{4}$$

Here $P_k(x)$ $(k = 0, 1, \cdots, m)$ and $f(x)$ are functions defined on $a \leq c \leq b$; the real coefficients $a_{i,j}, b_{i,j}, c_{i,j}, \lambda_i$ are appropriate constants. N number shows till which term of the series it will be expansioned and $y^{(n)}(c)$ are the Taylor coefficients to be determined. We use collocation points at defined interval of problem to find the Taylor coefficients.

$$a = x_0 < x_1 < \cdots < x_N = b$$

and the collocation points,

$$x_i = a + i\frac{b-a}{N},\ i = 0, 1, 2, \cdots, N$$

then we can put series (3) in the matrix form

$$[y(x)] = X M_0 A \tag{5}$$

where

$$X = \begin{bmatrix} 1 & (x-c) & (x-c)^2 & \cdots & (x-c)^n \end{bmatrix}$$

$$A = \begin{bmatrix} y^{(0)}(c) & y^{(1)}(c) & y^{(2)}(c) & \cdots & y^{(n)}(c) \end{bmatrix}^t$$

$$M_0 = \begin{bmatrix} \frac{1}{0!} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{1!} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{2!} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{N!} \end{bmatrix}$$

Firstly, we substitute x_i Taylor collocation points in Eq.(5),

$$\begin{aligned} &[y(x_i)] = X_i M_0 A;\ i = 0, 1, \cdots, N \\ &X_i = \left[\begin{array}{cccccc} 1 & (x_i - c) & (x_i - c)^2 & \cdots & (x_i - c)^n \end{array}\right] \end{aligned} \tag{6}$$

and,

$$\begin{aligned} &[y(x_0)] = X_0 M_0 A \\ &[y(x_1)] = X_1 M_0 A \\ &\vdots \\ &[y(x_N)] = X_N M_0 A \end{aligned}$$

$$Y^{(0)} = C M_0 A \tag{7}$$

where the matrices form $Y^{(0)}$ and C,

$$\begin{aligned} Y^{(0)} &= \left[\begin{array}{ccccc} y(x_0) & y(x_1) & y(x_2) & \cdots & y(x_N) \end{array}\right]^t \\ C &= \left[\begin{array}{ccccc} x_0 & x_1 & x_2 & \cdots & x_N \end{array}\right]^t \\ &= \begin{bmatrix} 1 & (x_0 - c) & (x_0 - c)^2 & \cdots & (x_0 - c)^N \\ 1 & (x_1 - c) & (x_1 - c)^2 & \cdots & (x_1 - c)^N \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (x_N - c) & (x_N - c)^2 & \cdots & (x_N - c)^N \end{bmatrix} \end{aligned}$$

$y^{(k)}(c)$ are the matrix forms of the derivates functions.

$$\left[y^{(k)}(x)\right] = X M_k A,\ k = 0, 1 \ldots, m \leq N \tag{8}$$

we substitute x_i Taylor collocation points in Eq.(8),

$$Y^{(k)} = C M_k A,\ k = 0, 1 \ldots, m \leq N \tag{9}$$

where matrix $Y^{(k)}$,

$$Y^{(k)} = \left[\begin{array}{ccccc} y^{(k)}(x_0) & y^{(k)}(x_1) & y^{(k)}(x_2) & \cdots & y^{(k)}(x_N) \end{array}\right]^t$$

we substitute x_i Taylor collocation points in Eq.(2),

$$P_0 Y^{(0)} + P_1 Y^{(1)} + \cdots + P_m Y^{(m)} = F \text{ or } \sum_{k=0}^{m} P_k Y^{(k)} = F \tag{10}$$

where matrices P_k and F for $k = 0, 1, \cdots, m \leq N$,

$$P_k = \begin{bmatrix} P_k(x_0) & 0 & \cdots & 0 \\ 0 & P_k(x_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_k(x_N) \end{bmatrix}_{(N+1)\times(N+1)},$$

$$F = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_N) \end{bmatrix}_{(N+1)\times 1}$$

we substitute $Y^{(k)}$ Taylor collocation points in Eq.(9),

$$\left\{ \sum_{k=0}^{m} P_k C M_k \right\} A = F \tag{11}$$

As the above mentioned matrices aren't easy to calculate, we can show the matrices by calculating via Maple 12. The procedures of this matrices in Maple 12 and an example can be written as[5],

Procedure 1[6]

```
Mmatrix:= proc (N,m)
local i,j,k,f,M;
for k from 0 to 5 do
f[k]:=(i,j) -> piecewise(i=j-k,1/(i-1)!):
M[k]:=matrix(N,N,f[k]
od:
eval(M[m]):
end:
```

where, N is dimension and m is supscript.

Example :

```
> M0:=Mmatrix(5,0);
```

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{24} \end{bmatrix}$$

Procedure 2[6]

```
Pmatrix:= proc (N,a,b,p)
local i,j,k,M,P,g,h;
with(linalg):
for k from 0 to N do
h[k]:=a+k*(b-a)/(N-1):od:
for k from 1 to N do
g[k]:=(i,j) -> piecewise(i=j,subs(x=h[i-1],p)):
P:=matrix(N,N,g[k]):od:
eval(P):end:
```

where, N is dimension and p is a P(x) polynomial.

Example :

> **P0:=Pmatrix(5,0,1,x);**

$$P0 := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Procedure 3[6]

```
Cmatrix:= proc (N,a,b)
local i,j,k,f,M,x,h,g;
with(linalg):
for k from 1 to N do
x[k]:=a+(k-1)*(b-a)/(N-1):od:
for k from 1 to N do
f[k]:=(i,j) -> simplify((x[k]-a)^(j-1)):
h[k]:= matrix(1,N,f[k]):
od:
g[1]:=h[1]:
for k from 1 to N-1 do
g[k+1]:=linalg[stackmatrix](g[k],h[k+1]):
od:
Eval(g[N]):end:
```

where, N is dimension.

Example:
> **C0:=Cmatrix(5,0,1);**

$$C0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & \frac{1}{4} & \frac{1}{16} & \frac{1}{64} & \frac{1}{256} \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} \\ 1 & \frac{3}{4} & \frac{9}{16} & \frac{27}{64} & \frac{81}{256} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Procedure 4[6]

```
Hmatrix:= proc (N,a,b)
local i,j,k,f,M,x,h,g;
f:=(i,j) -> simplify((b-a)^(j-1)):
h:= matrix(1,N,f):
eval(h):
end:
```

where N is dimension

Example:
> **H:=Hmatrix(5,0,0); L:=Hmatrix(5,0,1/2);**

$$H := [1 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$L := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

Procedure 5[6]

```
Fmatrix:= proc (N,a,b,f)
local i,j,k,h,g,F;
for k from 1 to N do
h[k]:=a+(k-1)*(b-a)/(N-1):od:
for k from 1 to N do
g[k]:=(i,j) -> simplify(subs(x=h[i],f)):
F:=matrix(N,1,g[k]):
od:eval(F):end:
```

where, N is dimension and f is f(x) function.

Example:

> **F:=Fmatrix(5,0,1,x^2);**

$$F := \begin{bmatrix} 0 \\ \frac{1}{16} \\ \frac{1}{4} \\ \frac{9}{16} \\ 1 \end{bmatrix}$$

Procedure 6[6]

```
Answer:= proc (N,A::matrix)
local i,j,k,f,T,C;
f:=(i,j) -> x^(j-1)/(j-1)!:
T:=matrix(1,N+1,f);C:=multiply(T,A);eval(C):
end:
```

where, N is dimension. It calculates equalence of A matrix in the Taylor serie which was calculated before.

Example:

> **answer:=Answer(4,A);**

$$answer := \left[6 + x + \frac{241973}{481381} x^2 + \frac{74144}{481381} x^3 + \frac{88240}{1444143} x^4 \right]$$

where, we indicate some Maple command,[**6**].

> **with(linalg):A:= matrix(3,3,[1,2,3,4,5,6,7,8,9]);**

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

> **delrows(A, 2..3);**

$$[1 \quad 2 \quad 3]$$

It is used to delete 2. and 3. rows of A matrix.

> **delcols(A, 1..1);**

$$\begin{bmatrix} 2 & 3 \\ 5 & 6 \\ 8 & 9 \end{bmatrix}$$

It is used to delete a column of A matrix.

> **with(linalg): A:= matrix(2,2,[1,2,3,4]): B:= matrix(2,2,[5,6,7,8]);**

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B := \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

> **stackmatrix(A,B);**

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

It is used for putting the A and B matrices one under the other.

2. Application

$$y'' - \frac{\sin x}{\cos x} y' = -2\sin x,\ \ y(0) = 0, y'(0) = 1$$

The exact solution of equation is $y(x) = \sin x$. Now, let's solve this problem with the mentioned method and Maple procedure [6].

$$y(x) = \sum_{n=0}^{4} \frac{y^n(c)}{n!}(x-c)^n,\ \ 0 \le x, c \le 1$$

we write Taylor collocation points for N=4,

$$x_0 = 0, x_1 = \frac{1}{4}, x_2 = \frac{1}{2}, x_3 = \frac{3}{4}, x_4 = 1$$

and,

$$P_1(x) = -\frac{\sin(x)}{\cos(x)}, P_2(x) = 1, f(x) = -2\sin(x)$$

the matrix equation for these functions,

$$\{P_2CM_2 + P_1CM_1\}\, A = F$$

and the equations of condition is created,

$$\tilde{U}_0 = [U_0; \lambda_0] = \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & ; & 0 \end{array}\right]$$
$$\tilde{U}_1 = [U_1; \lambda_1] = \left[\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & ; & 0 \end{array}\right]$$

> **P1:=Pmatrix(5,0,1,sin(x)/cos(x));**

$$P1 := \begin{bmatrix} 0. & 0 & 0 & 0 & 0 \\ 0 & 0.2553419213 & 0 & 0 & 0 \\ 0 & 0 & 0.5463024898 & 0 & 0 \\ 0 & 0 & 0 & 0.9315964599 & 0 \\ 0 & 0 & 0 & 0 & 1.557407725 \end{bmatrix}$$

> **C:=Cmatrix(5,0,1);**

$$C := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & \frac{1}{4} & \frac{1}{16} & \frac{1}{64} & \frac{1}{256} \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} \\ 1 & \frac{3}{4} & \frac{9}{16} & \frac{27}{64} & \frac{81}{256} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

> **M2:=Mmatrix(5,2);**

$$M2 := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

> **M1:=Mmatrix(5,1);**

$$M1 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

> **M0:=Mmatrix(5,0);**

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{24} \end{bmatrix}$$

> **F:=Fmatrix(5,0,1,-2*sin(x));**

$$F := \begin{bmatrix} 0. \\ -0.4948079186 \\ -0.9588510772 \\ -1.363277520 \\ -1.682941970 \end{bmatrix}$$

> **with(linalg): evalm(P1&*C&*M1);**

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0. & 0.2553419213 & 0.06383548032 & 0.007979435040 & 0.0006649529200 \\ 0. & 0.5463024898 & 0.2731512449 & 0.06828781120 & 0.01138130187 \\ 0. & 0.9315964599 & 0.6986973449 & 0.2620115044 & 0.06550287608 \\ 0. & 1.557407725 & 1.557407725 & 0.7787038625 & 0.2595679542 \end{bmatrix}$$

> **evalm(C&*M2);**

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{32} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{8} \\ 0 & 0 & 1 & \frac{3}{4} & \frac{9}{32} \\ 0 & 0 & 1 & 1 & \frac{1}{2} \end{bmatrix}$$

```
> A:=evalm(evalm(C&*M2)-evalm(P1&*C&*M1));
```

$$A := \begin{bmatrix} 0. & 0. & 1. & 0. & 0. \\ 0. & -0.2553419213 & 0.9361645197 & 0.2420205650 & 0.03058504708 \\ 0. & -0.5463024898 & 0.7268487551 & 0.4317121888 & 0.1136186981 \\ 0. & -0.9315964599 & 0.3013026551 & 0.4879884956 & 0.2157471239 \\ 0. & -1.557407725 & -0.557407725 & 0.2212961375 & 0.2404320458 \end{bmatrix}$$

```
> delrows(A, 4..5);
```

$$\begin{bmatrix} 0. & 0. & 1. & 0. & 0. \\ 0. & -0.2553419213 & 0.9361645197 & 0.2420205650 & 0.03058504708 \\ 0. & -0.5463024898 & 0.7268487551 & 0.4317121888 & 0.1136186981 \end{bmatrix}$$

```
> h1:=Hmatrix(5,0,0);
```

$$h1 := [1 \quad 0 \quad 0 \quad 0 \quad 0]$$

```
> evalm(h1&*M0);
```

$$[1 \quad 0 \quad 0 \quad 0 \quad 0]$$

```
> evalm(h1&*M1);
```

$$[0 \quad 1 \quad 0 \quad 0 \quad 0]$$

```
> delrows(F, 4..5);
```

$$\begin{bmatrix} 0. \\ -0.4948079186 \\ -0.9588510772 \end{bmatrix}$$

> **W:=stackmatrix(delrows(A,4..5),evalm(h1&*M0),evalm(h1&*M1));**

$$W := \begin{bmatrix} 0. & 0. & 1. & 0. & 0. \\ 0. & -0.2553419213 & 0.9361645197 & 0.2420205650 & 0.03058504708 \\ 0. & -0.5463024898 & 0.7268487551 & 0.4317121888 & 0.1136186981 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

> **f:=stackmatrix(delrows(F, 4..5),[0],[1]);**

$$f := \begin{bmatrix} 0. \\ -0.4948079186 \\ -0.9588510772 \\ 0 \\ 1 \end{bmatrix}$$

> **p:=linsolve(W,f);**

$$p := \begin{bmatrix} 0. \\ 1 \\ 0. \\ -1.020698925 \\ 0.2473147464 \end{bmatrix}$$

> **answer:=Answer(4,p);**

$$answer := \left[x - 0.1701164875x^3 + 0.01030478110x^4\right]$$

$$y(x) := x - 0.1701164875x^3 + 0.01030478110x^4$$

3. Conclusion

This study is about the differential equations which don't have analitical solutions or have so difficult and time-consuming solutions. Firstly,we obtain matrix form depending on the values in collocation points of the familiar coefficient functions and unknown function and its derivatives in differential equations,finite Taylor series expansion. Then,the equation is converted a matrix equation with Taylor coefficient by substituting this matrix form. Taylor collocation method can solve only the result matrix equations which corresponding the linear algebraic system. So the solution can not find for nonlinear systems.

But this method gives to approach result to analytical solution in linear equations and it can easily solve with Maple procedures.

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