

An Application of Determining the Best Tax Ratios by Goal Programming

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Abstract. Numerous procedures are used to solve multiple criteria decision-making problems. Among these the most frequently used procedure is goal programming, which is also the selected procedure for this study. It will be used to calculate the best tax ratios for the city of Isparta, a daily problem.

Key words: Linear programming, goal programming.

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1. Introduction

Whether it is in day to day life or in their careers, people carry on their lives by making decisions, some of them insignificant and some of them crucial. To make a decision in their favor a person will always have more than one choice. In time due to the difference between goals and preferences, decisions made with one goal in mind gradually transform into a multiple criteria goal orientated system. Multiple criteria decision – making is required in situations where there is more than one goal or there are numerous ways to reach the set goal. Even though procedures of this type have gained a lot of ground in recent years, procedures that have been tried, tested and used by researchers for many years are the most popular and superior. It has been proven that knowing linear algebra, the simplex method and various computer programmes help in understanding multiple criteria decision-making procedures. Once the decision maker eliminates these necessities, solving the maximization and minimization problem will help them choose at the decision making stage. Multiple criteria decision-making problems are classified according to what level their application area, goals, constraints relate to the decision maker. In practice the most frequently used is The Goal Programming procedure as it has a very wide area of use [8].

Goal programming is a special advanced linear programme used to resolve decision-making problems regardless of whether the issue is multiple goal problems with multiple sub goals or one goal with multiple sub goals. Similar to linear programming, there are no sizing constraints on the goal function in goal programming. Instead of just expressing goal benchmarks as minimum and maximum within given constraints, goal programming is a technique which also focuses on keeping deviations within goals to a minimum. Taking into account the inadequacy of the comprehensive and resilience constrictions for linear programming Charnes and Cooper first suggested this technique in 1950 [1]. An article written by Charnes in 1955 initiated goal programming. The term goal programming was first used in 1961 as a result of a Charnes and Cooper study, the subject of which was handling a wider version of restricted regression, which also included comprehensive linear models. Charnes and Cooper defined three configurations for goal programming in order to keep undesired variations to a minimum. At the end of 1960 software was developed for algorithm application to assist in solving goal programming problems. Goal programming was further developed by Ignizio [4], Lee [11], Tamiz [12] and Romero and Rehman [5].

2. Findings

The City Hall of Isparta, a city with a population of 201669, is endeavoring to create a fair regulation for the taxes it collects. The estimated income from taxes is based on the figures from Table 1. Tax bases for the following taxes and fees have been included in the table illustrated below: facilities, building land, real estate, sanitation, communication, entertainment, fire insurance, advertising, construction, slaughterhouse examination and inspection, toll, trading license, measuring and weighting equipment examination, work permit for ferial days, brokerage, wholesale food markets and residential usage license.

Table 1. Estimated annual tax bases for taxes and fees.

Types of Taxes and Fees	Tax Bases (TL)
Facilities Tax	2.140.726.020
Building Land Tax	140.180.473, 3
Real Estate Tax	19.999.750
Sanitation Tax	23.361.547, 17
Communication Tax	12.885.875
Entertainment Tax	4.241.556
Fire Insurance Tax	1.727.498, 7
Advertising Tax	62.858.835
Construction Fees	477.323.500
Slaughterhouse Examination and Inspection Fees	1.493.172, 8
Toll Charges	1.747.526, 5
Trading License Fees	6.563.415
Measuring and Weighting Equipment Fees	254.520
Work Permit (for ferial days) Fee	345.500
Brokerage Fee	6.059.978
Wholesale Food Market Legal Fee	13.999.762, 5
Residential Usage License Fee	155.963.043, 3

Assuming that the City Hall of Isparta City has the following goals we set up the mathematical model of the problem of determining the best tax ratios.

- G* 1. In order to meet the City's financial need tax income needs to be at least 6.500.000 (Turkish Lira.).
- G* 2. Facilities tax cannot exceed 17% of the overall taxes.
- G* 3. Sanitation tax cannot exceed 14% of the overall taxes.
- G* 4. Communication tax cannot exceed 2% of the overall taxes.
- G* 5. Entertainment tax cannot exceed fire insurance tax.
- G* 6. Advertising tax cannot be less than the total income of entertainment tax, fire insurance tax and wholesale food market legal fees.
- G* 7. Residential usage license fee cannot exceed the total of facilities tax and building land tax.
- G* 8. Trading license fee cannot be less than the total of brokerage fees and slaughterhouse examination and inspection fees.
- G* 9. Communication tax cannot be more than five times the entertainment tax.
- G* 10. Work permit (for ferial days) fees cannot be less than three times the measuring and weighting equipment examination fee.

3. The Formation of the Mathematical Model

3.1. The Formulation of Goal Programming

Linear programming uses a mathematical model which explains the problem it deals with. The linearity property states that all functions within the mathematical model must be linear functions. Linear programming will choose the appropriate goal function (minimum or maximum) while adhering to variables and constraints. The linear programming model has four stages:

1. Determining decision variables,
2. Determining the goal function,
3. Determining constraints,
4. Sign constraint.

Mathematical software for linear programming model is as follows:

$$\begin{array}{ll}
 \text{Maximum } (z) & = c_1x_1 + c_2x_2 + \dots + c_nx_n \\
 \text{Constraints} & \\
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n & \leq b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n & \leq b_2 \\
 \dots & \dots \\
 a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n & \leq b_m \\
 \text{and} & \\
 x_1, x_2, \dots, x_n & \geq 0.
 \end{array}$$

[2].

As opposed to the traditional linear programming with a single goal function, goal programming determines goal deviations with regards to constraints, priorities goals and restricts all function variables to the same unit of measurement. Goal programming also keeps deviations from the goal to a minimum. Goal programming is formed of four components; decision variables, deviation variables, system constraints and an goal function. Deviant variables are shown in two dimensions; positive and negative. These deviant variables form the goal function:

$$\begin{array}{ll}
 d_i^+ & : \text{ Positive deviant variable} \\
 d_i^- & : \text{ Negative deviant variable.}
 \end{array}$$

In accordance with the positive and negative deviant variables the formulation of goal programming is as follows:

$$\begin{array}{ll}
 \text{Min } z & = \sum_{i=1}^m (d_i^+ + d_i^-) \\
 \sum_{j=1}^n a_{ij}x_j - d_i^+ + d_i^- & = b_i \\
 d_i^+, d_i^-, x_j & \geq 0 \\
 i = 1, \dots, m & , \quad j = 1, \dots, n.
 \end{array}$$

[4].

Positive deviation and negative deviation can not be presented simultaneously, therefore, one or both of the deviant variables will be zero. Once the unwanted deviants variables are defined the formulation of goal programming can commence. Only one of the variables will be minimized by the decision-maker [3].

3.2. Composing The Model of The Problem

Once the decision variables for the taxes and fees are determined, the goals are defined. The decision variables in our model are:

- x_1 : Facilities tax
 - x_2 : Building land tax
 - x_3 : Real estate tax
 - x_4 : Sanitation tax
 - x_5 : Communication tax
 - x_6 : Entertainment tax
 - x_7 : Fire insurance tax
 - x_8 : Advertising tax
 - x_9 : Construction fees
 - x_{10} : Slaughterhouse examination and inspection fees
 - x_{11} : Toll charges
 - x_{12} : Trading license fees
 - x_{13} : Measuring and weighting equipment examination fees
 - x_{14} : Work permit (for ferial days) fees
 - x_{15} : Brokerage fees
 - x_{16} : Wholesale food market legal fees
 - x_{17} : Residential usage license fees.
- Goals are defined as:

$$\begin{aligned} 9200000 \leq & 2140726x_1 + 140180473.3x_2 + 19999750x_3 + 23361547.17x_4 \\ & 12885875x_5 + 4241556x_6 + 1727498.7x_7 + 62858835x_8 \\ & 477323500x_9 + 1493172.8x_{10} + 1747526.5x_{11} + 6563415x_{12} \\ & +254520x_{13} + 345500x_{14} + 6059978x_{15} + 13999762.5x_{16} \\ & +155963043.3x_{17}, \end{aligned}$$

$$\begin{aligned}
2140726020x_1 \leq & 0.17(2140726020x_1 + 140180473.3x_2 + 19999750x_3 \\
& + 23361547.17x_4 + 12885875x_5 + 4241556x_6 \\
& + 1727498.7x_7 + 62858835x_8 + 477323500x_9 \\
& + 1493172.8x_{10} + 1747526.5x_{11} + 6563415x_{12} \\
& + 254520x_{13} + 345500x_{14} + 6059978x_{15} \\
& + 13999762.5x_{16} + 155963043.3x_{17}),
\end{aligned}$$

$$\begin{aligned}
23361547.17x_4 \leq & 0.14(2140726x_1 + 140180473.3x_2 + 19999750x_3 \\
& + 23361547.17x_4 + 12885875x_5 + 4241556x_6 \\
& + 1727498.7x_7 + 62858835x_8 + 477323500x_9 \\
& + 1493172.8x_{10} + 1747526.5x_{11} + 6563415x_{12} \\
& + 254520x_{13} + 345500x_{14} + 6059978x_{15} \\
& + 13999762.5x_{16} + 155963043.3x_{17}),
\end{aligned}$$

$$\begin{aligned}
12885875x_5 \leq & 0.02(2140726x_1 + 140180473.3x_2 + 19999750x_3 \\
& + 23361547.17x_4 + 12885875x_5 + 4241556x_6 \\
& + 1727498.7x_7 + 62858835x_8 + 477323500x_9 \\
& + 1493172.8x_{10} + 1747526.5x_{11} + 6563415x_{12} \\
& + 254520x_{13} + 345500x_{14} + 6059978x_{15} \\
& + 13999762.5x_{16} + 155963043.3x_{17}),
\end{aligned}$$

$$4241556x_6 \leq 1727498.7x_7,$$

$$4241556x_6 + 1727498.7x_7 + 13999762.5x_{16} \leq 62858835x_8,$$

$$155963043.3x_{17} \leq 2140726020x_1 + 140180473.3x_2,$$

$$6059978x_{15} + 1493172.8x_{10} \leq 6563415x_{12},$$

$$12885875x_5 \leq (5 \cdot 4241556)x_6,$$

$$(3 \cdot 254520)x_{13} \leq 345500x_{14},$$

$$x_i \geq 0 \quad (1 \leq i \leq 17).$$

After arranging the goals above we get.

$$\begin{aligned}
& 2140726x_1 + 140180473.3x_2 + 19999750x_3 + 23361547.17x_4 \\
& + 12885875x_5 + 4241556x_6 + 1727498.7x_7 + 62858835x_8 \\
& + 477323500x_9 + 1493172.8x_{10} + 1747526.5x_{11} + 6563415x_{12} \\
& + 254520x_{13} + 345500x_{14} + 6059978x_{15} + 13999762.5x_{16} \\
& + 155963043.3x_{17} \geq 9200000, \\
& -1776802597x_1 + 2380680.46x_2 + 3399957.5x_3 + 3971463.019x_4 \\
& + 2190598.75x_5 + 721064.52x_6 + 293674.779x_7 + 10686001.95x_8 \\
& + 81144995x_9 + 253839.376x_{10} + 297079.505x_{11} + 1115780.55x_{12} \\
& + 43268.4x_{13} + 58735x_{14} + 1030196.26x_{15} + 2379959.625x_{16} \\
& + 26513717.36x_{17} \geq 0, \\
& 41958229.99x_1 + 19625266.26x_2 + 2799965x_3 - 20090930.57x_4 \\
& + 1804022.5x_5 + 593817.84x_6 + 241849.818x_7 + 8800236.9x_8 \\
& + 66825290x_9 + 209044.192x_{10} + 244653.71x_{11} + 918878.1x_{12} \\
& + 35632.8x_{13} + 48370x_{14} + 848396.92x_{15} + 1959966.75x_{16} \\
& + 21834826.06x_{17} \geq 0, \\
& 42814520.4x_1 + 2803609.466x_2 + 399995x_3 + 467230.9434x_4 \\
& - 12628157.5x_5 + 84831.12x_6 + 34549.974x_7 + 1257176.7x_8 \\
& + 9546470x_9 + 29863.456x_{10} + 34950.053x_{11} + 131268.3x_{12} \\
& + 5090.4x_{13} + 6910x_{14} + 12119956x_{15} + 279995.25x_{16} \\
& + 3119260.866x_{17} \geq 0, \\
& 1727498.7x_7 - 4241556x_6 \geq 0, \\
& 62858835x_8 - 4241556x_6 - 1727498.7x_7 - 13999762.5x_{16} \geq 0, \\
& 2140726020x_1 + 140180473.3x_2 - 155963043.3x_{17} \geq 0, \\
& 6563415x_{12} - 6059978x_{15} - 1493172.8x_{10} \geq 0, \\
& 21207780x_6 - 12885875x_5 \geq 0, \\
& 345500x_{14} - 763560x_{13} \geq 0, \\
& x_i \geq 0 \quad (1 \leq i \leq 17).
\end{aligned}$$

The lower and upper bounds of tax rates for our model are as the following:

$$\begin{aligned}
0.001 &\leq x_1 \leq 0.0011, \\
0.006 &\leq x_2 \leq 0.06, \\
0.01 &\leq x_3 \leq 0.011, \\
0.003 &\leq x_4 \leq 0.03, \\
0.1 &\leq x_5 \leq 0.11, \\
0.001 &\leq x_6 \leq 0.01, \\
0.025 &\leq x_7 \leq 0.0275, \\
0.004 &\leq x_8 \leq 0.04, \\
0.01 &\leq x_9 \leq 0.011, \\
0.002 &\leq x_{10} \leq 0.02, \\
0.0015 &\leq x_{11} \leq 0.00165, \\
0.0003 &\leq x_{12} \leq 0.003, \\
0.001 &\leq x_{13} \leq 0.0011, \\
0.001 &\leq x_{14} \leq 0.01, \\
0.003 &\leq x_{15} \leq 0.0033, \\
0.002 &\leq x_{16} \leq 0.02, \\
0.15 &\leq x_{17} \leq 0.165.
\end{aligned}$$

3.3. The solution of the model

The problem is solved by using the "fgoalattain" mat lab command within the "optimization toolbox". The mathematical representation of goal programming in Mat lab context is:

$$\begin{aligned}
\min_{x, \gamma} \quad & F(x) - w * \gamma \leq h \\
& c(x) \leq 0, \quad d(x) = 0 \\
& A * x \leq b, \quad B * x = m \\
& D_{down} \leq x \leq D_{up}.
\end{aligned}$$

The equations stated above define a comprehensive (contains more than one goal function) optimization problem. In the equations; x , w (gravity vector), h (goal vector), b , m , D_{up} , D_{down} values are vectors, A and B values are matrices, $c(x)$, $d(x)$ and $F(x)$ variables define the functions. These three functions may not be linear. The fgoalattain command makes an effort to reach the goal by using the F goal function and the " h " (goal) vector.

According to the approximate results of the problem; function values reached their predetermined goals without deviation, x values defined the correct tax ratios in order to achieve the goals to be reached successfully. The success of the calculations is underlined with the control value, which is 1. The problem has ten goals. As the access factor is close to zero it can be speculated that objectives have nearly been reached.

4. Results

The tax ratios, having solved the model in line with the determined goals, are as in the following Table 2:

Table 2. The tax ratios for the taxes and fees.

Types of taxes and fees	Tax Ratios
x_1	0.0010
x_2	0.0060
x_3	0.0100
x_4	0.0030
x_5	0.1000
x_6	0.0010
x_7	0.0250
x_8	0.0040
x_9	0.0100
x_{10}	0.0020
x_{11}	0.0015
x_{12}	0.0003
x_{13}	0.0011
x_{14}	0.0010
x_{15}	0.0030
x_{16}	0.0020
x_{17}	0.1500

The x values are the tax ratios that should be used for 2009. They will allow the City Hall to determine the tax ratios for next year appropriately.

By using the goal programming procedure, a comprehensive decision-making application, The City Hall will be able to determine its tax ratios both in the short term and the long term by setting its goals. While the City Hall determines how they will reach their set goals and what their tax bases should be in order to achieve these goals, they will also be able to analyze results obtained from the solutions. The City Hall will have the opportunity to choose the best tax bases and fee types for the coming year in line with their policies just by evaluating the results. The aim is not just to reach set goals. Sometimes a model that satisfies all goals may not relate to the City Hall politics.

References

1. Charnes, A., Cooper W. W., (1961): Management Models and Industrial Application of Linear Programming, (John Wiley and Sons, New York, Vol 1).
2. Dantzig, George B., (1974): On a Convex Programming Problem of Rozanov, Applied Mathematics of Optimization, 1,2, 189-192.

3. Güneş, M., Umarusman, N. (1994): Bir Karar Destek Aracı Bulanık Hedef Programlama ve Yerel Yönetimlerde Vergi Optimizasyonu Uygulaması, *Review of Social Economic & Business Studies*, 2, 244-255.
4. Ignizio, James P., (1985): *Introduction to Linear Goal Programming*, (Sage Publication, California).
5. Lee, S. M., Moore, L., (1975): *Introduction to Decision Science*, (Petrocelli / Charter, New York).
6. Romero, C., Rehman, T., (1984): Goal Programming and Multiple Criteria Decision Making in Farm Planning, *Journal of Agricultural Economics*, 35, 2, 177-190.
7. Romero, C., (2001): Extended Lexicographic Goal Programming, A Unifying Approach, *Omega*, *The International Journal of Management Science*, 29, 63-67.
8. Şahiner, A., Acar S., (2008): Süleyman Demirel Üniversitesi Kampüsünde Güvenlik Birimi Yerleştirilmesi Optimizasyonu, *Süleyman Demirel Üniversitesi, Fen Bilimleri Enstitüsü Dergisi*, 12-3, 168-172.
9. Tamiz, M., (1996): *Multi-Objective Programming and Goal Programming*, (Springer, Berlin).
10. Turanlı M., Köse A., (2005): Doğrusal Hedef Programlama Yöntemi ile Türkiye'deki Sigorta Şirketlerinin Performanslarının Değerlendirilmesi, *İstanbul Ticaret Üniversitesi Fen Bilimleri Dergisi*, 4, 7, 19-39.
11. Öztürk, A.,(2005): *Yöneylem Araştırması*, (Ekin Kitabevi, Bursa).
12. Youness, E. A., (1995): A Direct Approach for Finding all Efficient Solutions for Multiobjective Programming Problems, *European Journal of Operational Research*, 81, 440-443.