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Scattering by a Moving Circular Cylinder in Hertzian Electrodynamics

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Abstract. We provide a general formulation of electromagnetic scattering by an arbitrarily moving material object in the context Hertzian Electrodynamics, which is followed by applications to 2-D canonical problems involving perfect electric conductor and dielectric circular cylinders under plane wave incidence for various modes (uniform, harmonic, rotational) of motion.

Key words: Maxwell equations; Moving media; Hertz equations; Continuum mechanics; Frame indifference; Progressive derivatives.
2000 Mathematics Subject Classification: 78A25, 35Q60, 74A05.

1. Introduction

The present work is structured as a sequel of [1] where we reviewed and extended certain aspects of the mathematical foundations, axiomatic structure and principles of Hertzian Electrodynamics (HE). The introduction of a commutative property of the comoving time derivative operator in Theorem A.4 in Appendix of [1] and the corresponding Hertzian wave equations helps one construct the associated boundary value problem for rigid bodies in arbitrary Euclidean motion. In that context in Section 2 we provide the general field and boundary relations for electromagnetic scattering by an arbitrarily moving material object. Specifically, we consider the scenario in Figure 1 where, according to an observer in Cartesian reference configuration $Ox_1x_2x_3t$, the incident electromagnetic wave with fields $(\vec{E}_{inc}(\vec{r};t), \vec{H}_{inc}(\vec{r};t))$ and sources $(\rho_{Tx}(\vec{r};t), \vec{J}_{Tx}(\vec{r};t))$ generated by a stationary transmitter in an ambient medium I is impinging on an object occupying a region D and in arbitrary relative motion with instantaneous velocity $\vec{\mathrm{v}}(\vec{r};t)$.

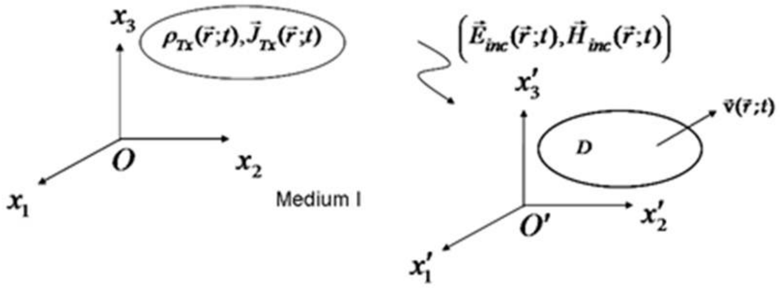

Figure 1. An illustration of a scattering problem

This is followed in the subsequent sections by applications to 2-D canonical problems involving perfect electric conductor (PEC) and dielectric circular cylinders under plane wave incidence for various modes (uniform, harmonic, rotational) of motion. A theoretical comparison of similar results using the principle of special or general space-time covariance is mentioned to in Concluding Remarks.
The reader is assumed to be already familiar with the terminology, definitions, postulates, theorems, etc. in [1] so that many of them shall not be repeated herein for practical reasons. In particular we shall frequently use the terms "E-frame" and "L-frame" as abbreviations of Eulerian and Lagrangian frames from fluid mechanics for denoting reference (spatial) and current (material) configurations for brevity.

2. The General Formulation of a Scattering Problem

When the coordinate transformations that describe the motion of the moving object is given, one can solve the corresponding scattering problem from an isolated moving body formally by "frame hopping"[1] following the steps below:

1. Map the incoming field from E- to L-frame

2. Solve the scattered field from the associated boundary value problem in L-frame

3. Map the scattered field back from L- to E-frame

In constructing the boundary value problem in L-frame, the corresponding spatial/temporal jump and edge conditions are obtained from the distributional investigation of the field equations along with complementary conditions such as radiation condition, periodicity, boundedness, *etc.*

2.1. The Incoming Wave

In E-frame the incident fields satisfy the Maxwell equations of stationary media (2.1a,b)

$$\operatorname{curl} \vec{E}_{inc}(\vec{r};t) + \frac{\partial}{\partial t}\vec{B}_{inc}(\vec{r};t) = \vec{0}, \ \operatorname{curl} \vec{H}_{inc}(\vec{r};t) - \frac{\partial}{\partial t}\vec{D}_{inc}(\vec{r};t) = \vec{J}_{Tx}(\vec{r};t)$$

[1] A methodology first employed by Van Bladel for relativistic scattering problems (cf. [2]).

(2.1c,d) $$\operatorname{div}\vec{D}_{inc}(\vec{r};t)=\rho_{Tx}(\vec{r};t),\ \operatorname{div}\vec{B}_{inc}(\vec{r};t)=0$$

Let us assume medium I simple and lossless with constitutive parameters (ε,μ). Then the incident time domain (& phasor) fields in E-frame satisfy the stationary wave (& Helmholtz) equations

$$\left(lap-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\begin{array}{c}\vec{E}_{inc}(\vec{r};t)\\ \vec{H}_{inc}(\vec{r};t)\end{array}\right)=\left(\begin{array}{c}(1/\varepsilon)\operatorname{grad}\rho_{Tx}(\vec{r};t)\\ \vec{0}\end{array}\right),$$

$$\left(lap+k^2\right)\left(\begin{array}{c}\vec{E}_{inc}(\vec{r})\\ \vec{H}_{inc}(\vec{r})\end{array}\right)=\left(\begin{array}{c}(1/\varepsilon)\operatorname{grad}\rho_{Tx}(\vec{r})\\ \vec{0}\end{array}\right),$$

where $c=1/\sqrt{\mu\varepsilon}$ is the phase velocity and $k=\omega_{inc}/c$ is the wave number with angular frequency ω_{inc} and time dependence taken as $\exp(-i\omega_{inc}t)$. For an observer in Cartesian L-frame $Ox_1'x_2'x_3't$, the object is stationary and the surrounding medium I is in relative motion with an instantaneous velocity $\vec{v}'(\vec{r}';t)$. Accordingly, in L-frame the incident fields satisfy the Hertzian field and wave equations

(2.3a) $$\operatorname{curl}'\vec{E}'_{inc}(\vec{r}';t)+\frac{\diamondsuit'}{\diamondsuit't}\vec{B}'_{inc}(\vec{r}';t)=\vec{0},$$

(2.3b) $$\operatorname{curl}'\vec{H}'_{inc}(\vec{r}';t)-\frac{\diamondsuit'}{\diamondsuit't}\vec{D}'_{inc}(\vec{r}';t)=\vec{J}'_{Tx}(\vec{r}';t)$$

(2.3c,d) $$\operatorname{div}'\vec{D}'_{inc}(\vec{r}';t)=\rho'_{Tx}(\vec{r}';t),\ \operatorname{div}'\vec{B}'_{inc}(\vec{r}';t)=0$$

(2.4) $$\left(lap'-\frac{1}{c^2}\frac{\diamondsuit'^2}{\diamondsuit'^2}\right)\left(\begin{array}{c}\vec{E}'_{inc}(\vec{r}';t)\\ \vec{H}'_{inc}(\vec{r}';t)\end{array}\right)=\left(\begin{array}{c}(1/\varepsilon)\,grad'\rho'_{Tx}(\vec{r}';t)\\ \vec{0}\end{array}\right).$$

In (2.3) the comoving time derivative of a vector $\vec{A}'_{inc}(\vec{r}';t)$ is given by

(2.5) $$\frac{\diamondsuit'}{\diamondsuit't}\vec{A}'_{inc}=\frac{\partial}{\partial t}\vec{A}'_{inc}+\vec{v}'\cdot\operatorname{grad}'\vec{A}'_{inc}-\vec{A}'_{inc}\cdot\operatorname{grad}'\vec{v}'+\vec{A}'_{inc}\operatorname{div}'\vec{v}'.$$

When the incident fields and sources in L-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega'_{inc}t)$, then their phasors satisfy the reduced field equations (see [1], Section 7)

(2.6a,b)

$$\operatorname{curl}'\vec{E}'_{inc}(\vec{r}')-i\omega_{inc}\vec{B}'_{inc}(\vec{r}')=\vec{0},\operatorname{curl}'\vec{H}'_{inc}(\vec{r}')+i\omega_{inc}\vec{D}'_{inc}(\vec{r}')=\vec{J}'_{Tx}(\vec{r}')$$

(2.6c,d) $$\operatorname{div}'\vec{D}'_{inc}(\vec{r}')=\rho'_{Tx}(\vec{r}'),\ \operatorname{div}'\vec{B}'_{inc}(\vec{r}')=0$$

and the Helmholtz equations

(2.7) $$\left(lap'+k^2\right)\left(\begin{array}{c}\vec{E}'_{inc}(\vec{r}')\\ \vec{H}'_{inc}(\vec{r}')\end{array}\right)=\left(\begin{array}{c}(1/\varepsilon)\operatorname{grad}'\rho'_{Tx}(\vec{r}')\\ \vec{0}\end{array}\right).$$

2.2. The Scattered Wave

Let us express the total field in space in E- and L-frames respectively as

$$(\vec{E}_{tot}, \vec{H}_{tot}) = \begin{cases} (\vec{E}_{inc}, \vec{H}_{inc}) + (\vec{E}_{sc}, \vec{H}_{sc}), \text{in medium I} \\ (\vec{E}_d, \vec{H}_d), \text{in region D} \end{cases}$$

and

$$(\vec{E}'_{tot}, \vec{H}'_{tot}) = \begin{cases} (\vec{E}'_{inc}, \vec{H}'_{inc}) + (\vec{E}'_{sc}, \vec{H}'_{sc}), \text{in medium I} \\ (\vec{E}'_d, \vec{H}'_d), \text{in region D} \end{cases} .$$

In L-frame of the scattered wave, i.e., with reference to the motion of region D, an L-observer senses the entire space (constituting the ambient *source free* medium I and region D) in motion with instantaneous velocity $-\vec{v}'(\vec{r}';t)$. Accordingly, the scattered fields in medium I satisfy the Hertzian equations

$$\text{(2.8a,b)} \quad \text{curl}'\,\vec{E}'_{sc}(\vec{r}';t) + \frac{\bar{\Diamond}'}{\bar{\Diamond}'t}\vec{B}'_{sc}(\vec{r}';t) = \vec{0},\ \text{curl}'\,\vec{H}'_{sc}(\vec{r}';t) - \frac{\bar{\Diamond}'}{\bar{\Diamond}'t}\vec{D}'_{sc}(\vec{r}';t) = \vec{0}$$

$$\text{(2.8c,d)} \qquad \text{div}'\,\vec{D}'_{sc}(\vec{r}';t) = 0, \text{div}'\,\vec{B}'_{sc}(\vec{r}';t) = 0$$

$$\text{(2.9)} \qquad \left(lap' - \frac{1}{c^2}\frac{\bar{\Diamond}'^2}{\bar{\Diamond}'t^2}\right)\begin{pmatrix} \vec{E}'_{sc}(\vec{r}';t) \\ \vec{H}'_{sc}(\vec{r}';t) \end{pmatrix} = \vec{0},$$

where the accompanying comoving time derivative of a vector $\vec{A}'_{sc}$ is defined as

$$\text{(2.10)} \qquad \frac{\bar{\Diamond}'}{\bar{\Diamond}'t}\vec{A}'_{sc} = \frac{\partial}{\partial t}\vec{A}'_{sc} - \vec{v}' \cdot \text{grad}'\,\vec{A}'_{sc} + \vec{A}'_{sc} \cdot \text{grad}'\,\vec{v}' - \vec{A}'_{sc}\,\text{div}'\,\vec{v}'.$$

When the scattered fields in L-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega'_{sc}t)$, then their phasors satisfy the Hertzian equations

$$\text{(2.11a,b)} \qquad \text{curl}'\,\vec{E}'_{sc}(\vec{r}') - i\omega_{inc}\vec{B}'_{sc}(\vec{r}') = \vec{0}, \text{curl}'\,\vec{H}'_{sc}(\vec{r}') + i\omega_{inc}\vec{D}'_{sc}(\vec{r}') = \vec{0}$$

$$\text{(2.11c,d)} \qquad \text{div}'\,\vec{D}'_{sc}(\vec{r}') = 0,\ \text{div}'\,\vec{B}'_{sc}(\vec{r}') = 0$$

$$\text{(2.12)} \qquad \left(lap' + k^2\right)\begin{pmatrix} \vec{E}'_{sc}(\vec{r}') \\ \vec{H}'_{sc}(\vec{r}') \end{pmatrix} = \vec{0}.$$

2.3. Total Field inside the Moving Object

In L-frame of the fields $(\vec{E}'_d, \vec{H}'_d)$ and sources $(\rho'_d, \vec{J}'_d)$ inside the moving object, the region D is sensed as stationary since the ambient medium I is observed as source-free. Therefore in region D the field equations of stationary media (2.13a,b)

$$\text{curl}'\,\vec{E}'_d(\vec{r}';t) + \frac{\partial}{\partial t}\vec{B}'_d(\vec{r}';t) = \vec{0},\ \text{curl}'\,\vec{H}'_d(\vec{r}';t) - \frac{\partial}{\partial t}\vec{D}'_d(\vec{r}';t) = \vec{J}'_d(\vec{r}';t)$$

$$\text{(2.13c,d)} \qquad \operatorname{div}' \vec{D}'_d(\vec{r}';t) = \rho'_d(\vec{r}';t), \ \operatorname{div}' \vec{B}'_d(\vec{r}';t) = 0$$

are satisfied. When the region D is simple with constitutive parameters $(\varepsilon_d, \mu_d, \sigma_d)$, (2.13) yield the stationary wave equations

$$\text{(2.14)} \quad \left(lap' - \frac{1}{c_d^2}\frac{\partial^2}{\partial t^2} - \sigma_d\mu_d \frac{\partial}{\partial t} \right) \begin{pmatrix} \vec{E}'_d(\vec{r}';t) \\ \vec{H}'_d(\vec{r}';t) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon_d) \operatorname{grad}' \rho'_d(\vec{r}';t) \\ \vec{0} \end{pmatrix},$$

with $c_d = 1/\sqrt{\mu_d\varepsilon_d}$. When the transmitted fields in L-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega'_d t)$, then their phasors satisfy the reduced field equations

$$\text{(2.15a,b)} \quad \operatorname{curl}' \vec{E}'_d(\vec{r}') - i\omega_{inc}\vec{B}'_d(\vec{r}') = \vec{0}, \ \operatorname{curl}' \vec{H}'_d(\vec{r}') + i\omega_{inc}\vec{D}'_d(\vec{r}') = \vec{J}'_d(\vec{r}')$$

$$\text{(2.15c,d)} \qquad \operatorname{div}' \vec{D}'_d(\vec{r}') = \rho'_d(\vec{r}'), \ \operatorname{div}' \vec{B}'_d(\vec{r}') = 0$$

and the Helmholtz equations

$$\text{(2.16)} \qquad \left(lap' + k_d^2\right) \begin{pmatrix} \vec{E}'_d(\vec{r}') \\ \vec{H}'_d(\vec{r}') \end{pmatrix} = \begin{pmatrix} (1/\varepsilon_d) \operatorname{grad}' \rho'_d(\vec{r}') \\ \vec{0} \end{pmatrix}$$

with $k_d^2 = \omega_{inc}^2\varepsilon_d\mu_d + i\omega_{inc}\sigma_d\mu_d$. For E-observer the field and wave equations (2.13), (2.14) read

$$\text{(2.17a,b)} \quad \operatorname{curl} \vec{E}_d(\vec{r};t) + \frac{\Diamond}{\Diamond t}\vec{B}_d(\vec{r};t) = \vec{0}, \ \operatorname{curl} \vec{H}_d(\vec{r};t) - \frac{\Diamond}{\Diamond t}\vec{D}_d(\vec{r};t) = \vec{J}_d(\vec{r};t)$$

$$\text{(2.17c,d)} \qquad \operatorname{div} \vec{D}_d(\vec{r};t) = \rho_d(\vec{r};t), \ \operatorname{div} \vec{B}_d(\vec{r};t) = 0$$

$$\text{(2.18)} \quad \left(lap - \frac{1}{c_d^2}\frac{\Diamond^2}{\Diamond t^2} - \sigma_d\mu_d \frac{\Diamond}{\Diamond t} \right) \begin{pmatrix} \vec{E}_d(\vec{r};t) \\ \vec{H}_d(\vec{r};t) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon_d) \operatorname{grad} \rho_d(\vec{r};t) \\ \vec{0} \end{pmatrix},$$

where the accompanying comoving time derivative of a vector $\vec{A}_d$ is defined as

$$\text{(2.19)} \qquad \frac{\Diamond}{\Diamond t}\vec{A}_d = \frac{\partial}{\partial t}\vec{A}_d + \vec{\mathrm{v}} \cdot \operatorname{grad} \vec{A}_d - \vec{A}_d \cdot \operatorname{grad} \vec{\mathrm{v}} + \vec{A}_d \operatorname{div} \vec{\mathrm{v}}.$$

When the transmitted fields in E-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega_{tr}t)$, then their phasors satisfy the reduced field equations

$$\text{(2.20a,b)} \qquad \operatorname{curl} \vec{E}_d(\vec{r}') - i\omega_{inc}\vec{B}_d(\vec{r}) = \vec{0}, \ \operatorname{curl} \vec{H}_d(\vec{r}) + i\omega_{inc}\vec{D}_d(\vec{r}) = \vec{J}_d(\vec{r})$$

$$\text{(2.20c,d)} \qquad \operatorname{div} \vec{D}_d(\vec{r}) = \rho_d(\vec{r}), \ \operatorname{div} \vec{B}_d(\vec{r}) = 0$$

and the Helmholtz equations

$$\text{(2.21)} \qquad \big(lap + k_d^2\big) \begin{pmatrix} \vec{E}_d(\vec{r}) \\ \vec{H}_d(\vec{r}) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon_d)\,\mathrm{grad}\,\rho_d(\vec{r}) \\ \vec{0} \end{pmatrix}.$$

When the incident fields in L-frame are not monochromatic but possess *arbitrary* waveforms which can be expressed as a superposition of monochromatic components in terms of a Fourier series or integral representation, then their each (discrete or continuous) component satisfies (2.7) individually, while similar arguments also hold in (2.12), (2.16) and (2.21).
It should be remarked that the wave numbers k, k_d remain invariant in E- and L-frames, while the derivation of the Hertzian wave equations (2.4) and (2.9) are restricted to rigid bodies in arbitrary Euclidean motion as expressed by (3.18) in [1].

2.4. Boundary Relations on the Moving Object

In the context of the scattering problems investigated in the subsequent sections we shall assume the enclosure $S = \partial D$ of the moving medium a simple interface, which might be a PEC or a dielectric interface supporting surface charges and currents $\rho'_S(\vec{r}'_S;t)$, $\vec{J}'_S(\vec{r}'_S;t)$. In these cases the distributional form of stationary field (Maxwell) equations in L- frame respectively read

$$\text{(2.22a)} \qquad \hat{n}' \times \Big[\vec{E}'_{inc}(\vec{r}'_S;t) + \vec{E}'_{sc}(\vec{r}'_S;t)\Big] = \vec{0}$$

$$\text{(2.22b)} \qquad \hat{n}' \times \Big[\vec{H}'_{inc}(\vec{r}'_S;t) + \vec{H}'_{sc}(\vec{r}'_S;t)\Big] = \vec{J}'_S(\vec{r}'_S;t)$$

$$\text{(2.22c)} \qquad \hat{n}' \cdot \Big[\vec{D}'_{inc}(\vec{r}'_S;t) + \vec{D}'_{sc}(\vec{r}'_S;t)\Big] = \rho'_S(\vec{r}'_S;t)$$

$$\text{(2.22d)} \qquad \hat{n}' \cdot \Big[\vec{B}'_{inc}(\vec{r}'_S;t) + \vec{B}'_{sc}(\vec{r}'_S;t)\Big] = 0$$

and

$$\text{(2.23a)} \qquad \hat{n}' \times \Big[\vec{E}'_{inc}(\vec{r}'_S;t) + \vec{E}'_{sc}(\vec{r}'_S;t)\Big] = \hat{n}' \times \vec{E}'_d(\vec{r}'_S;t)$$

$$\text{(2.23b)} \qquad \hat{n}' \times \Big[\vec{H}'_{inc}(\vec{r}'_S;t) + \vec{H}'_{sc}(\vec{r}'_S;t)\Big] = \hat{n}' \times \vec{H}'_d(\vec{r}'_S;t)$$

$$\text{(2.23c)} \qquad \hat{n}' \cdot \Big[\vec{D}'_{inc}(\vec{r}'_S;t) + \vec{D}'_{sc}(\vec{r}'_S;t)\Big] = \hat{n}' \cdot \vec{D}'_d(\vec{r}'_S;t)$$

$$\text{(2.23d)} \qquad \hat{n}' \cdot \Big[\vec{B}'_{inc}(\vec{r}'_S;t) + \vec{B}'_{sc}(\vec{r}'_S;t)\Big] = \hat{n}' \cdot \vec{B}'_d(\vec{r}'_S;t).$$

Along with constitutive relations and radiation, edge, tip, periodicity, boundedness etc. type complementary conditions, the associated boundary value problem can be solved *uniquely* to yield the L-fields $(\vec{E}'_{sc}, \vec{H}'_{sc})$ and $\left(\vec{E}'_d, \vec{H}'_d\right)$, whose maps also yield the E-fields $(\vec{E}_{sc}, \vec{H}_{sc})$ and $(\vec{E}_d, \vec{H}_d)$.

3. TE Plane Wave Scattering by a Moving Circular PEC Cylinder

In this section we consider the scattering of an incident monochromatic TE plane wave

(3.1a,b) $\quad \vec{E}_{inc}(\vec{r};t) = \hat{x}_3 e^{ik\hat{n}_{inc}\cdot\vec{r}} e^{-i\omega_{inc}t}, \ \vec{H}_{inc}(\vec{r};t) = (1/Z)\,\hat{n}_{inc} \times \vec{E}_{inc}(\vec{r};t)$

with $\hat{n}_{inc}\cdot\vec{r} = x_1\cos\alpha + x_2\sin\alpha$, $\alpha \in [0,\pi/2)$, $Z = \sqrt{\mu/\varepsilon}$ from an infinitely long circular PEC cylinder lying along x_3-axis, centered at origin and having radius a for three different modes of motion (see Figure 2). In cylindrical coordinates $x_1 = \rho\cos\phi$, $x_2 = \rho\sin\phi$ the incident electrical field is represented by

(3.1c) $$\vec{E}_{inc}(\vec{r};t) = \hat{x}_3 e^{ik\rho\cos(\phi-\alpha)} e^{-i\omega_{inc}t}$$

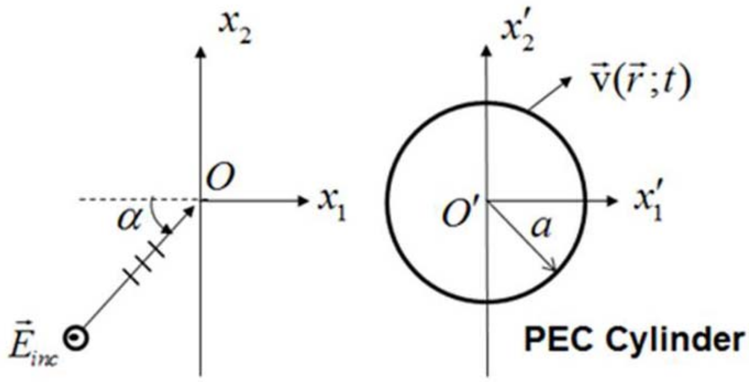

Figure 2. A cross section illustration of TE plane wave scattering by a moving circular PEC cylinder

3.1. Case I: Uniform Motion

Based on the coordinate transformations $x_1 = x'_1 + Gt$, $G = const$, $x_{2,3} = x'_{2,3}$; $\hat{x}_i = \hat{x}'_i$, $i = 1,2,3$, the incoming electrical field in L-frame reads

(3.2a) $$\vec{E}'_{inc}(\vec{r}';t) = \hat{x}'_3\, e^{ik\hat{n}'_{inc}\cdot\vec{r}'} e^{-i\omega'_{inc}t} = \hat{x}'_3\, e^{ik\rho'\cos(\phi'-\alpha)} e^{-i\omega'_{inc}t}$$

with $\omega'_{inc} = \omega_{inc}(1-\beta\cos\alpha)$, $\beta = G/c$, where we introduced the local cylindrical coordinates $\rho' = \sqrt{(x'_1)^2 + (x'_2)^2}$, $\phi' = \tan^{-1}(x'_2/x'_1)$. In virtue of the well known Bessel property

(3.2b)
$$e^{i\Omega \sin(\omega t)} = \sum_{-\infty}^{\infty} J_m(\Omega) e^{im\omega t}$$

the incident field can be expressed as a sum of hypothetical cylindrical modes as

(3.2c)
$$\begin{aligned}\vec{E}'_{inc}(\vec{r}';t) &= \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{inc}(\vec{r}')\, e^{-i\omega'_{inc}t} \\ &= \hat{x}'_3 \sum_{-\infty}^{\infty} J_m(k\rho')\, e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega'_{inc}t}.\end{aligned}$$

Based on the principle of superposition for sources and fields, which stem from the linear structure of field equations, the scattered field can be considered as a superposition of modes $\vec{E}'_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}';t) = \hat{x}'_3 \sum_{-\infty}^{\infty} E'^{(m)}_{sc}(\vec{r}')\, e^{-i\omega'^{(m)}_{sc}t}$, which satisfy the reduced boundary value problem

(3.3)
$$\begin{cases} \left(lap' + k^2\right) E'^{(m)}_{sc}(\vec{r}') = 0, \ \text{ in medium I} \\ \text{Boundary Condition :} \\ J_m(ka) e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega'_{inc}t} + E'^{(m)}_{sc}\left(\rho' = a, \phi'\right) e^{-i\omega'^{(m)}_{sc}t} = 0, \forall \phi', t \\ \text{Periodicity Condition : } E'^{(m)}_{sc}(\rho',\phi') = E'^{(m)}_{sc}(\rho',\phi'+2\pi), \ \forall \rho', \phi' \\ \text{Radiation Condition as } \rho' \to \infty \end{cases}$$

From (3.3) one uniquely solves $\omega'_{inc} = \omega'^{(m)}_{sc}$ and

(3.4)
$$E'^{(m)}_{sc}(\rho',\phi') = -\frac{J_m(ka)}{H^{(1)}_m(ka)} H^{(1)}_m(k\rho')\, e^{-im(\phi'-\alpha-\pi/2)}$$

to get

(3.5)
$$\begin{aligned}\vec{E}_{sc}(x_1,x_2;t) &= -\hat{x}_3 \sum_{-\infty}^{\infty} \tfrac{J_m(ka)}{H^{(1)}_m(ka)} H^{(1)}_m\left(k\sqrt{(x_1-Gt)^2 + x_2^2}\right) \\ &\times\, e^{-im\left(\tan^{-1}(x_2/(x_1-Gt))-\alpha-\pi/2\right)} e^{-i\omega_{inc}(1-\beta\cos\alpha)t}\end{aligned}$$

In the far field of the scatterer one can substitute the asymptotic formula

(3.6)
$$H^{(1)}_m(k\rho') \cong \sqrt{\frac{2}{\pi k\rho'}}\, e^{i\left(k\rho'-\pi/4-m\pi/2\right)}, k\rho' >> 1$$

into (3.4) to get

(3.7a)
$$\vec{E}'_{sc}(\rho',\phi';t) \cong \hat{x}'_3\, F'(\phi') \frac{e^{ik\rho'}}{\sqrt{k\rho'}} e^{-i\omega'_{inc}t}, k\rho' >> 1$$

with the scattering pattern

$$F'(\phi') = -e^{-i\pi/4}\sqrt{\frac{2}{\pi}}\sum_{-\infty}^{\infty}\frac{J_m(ka)}{H_m^{(1)}(ka)}\,e^{-im(\phi'-\alpha)}. \tag{3.7b}$$

The corresponding phasor modes of the magnetic field are derived as

$$\vec{H}'^{(m)}_{sc}(\vec{r}') = (1/i\mu\omega_{inc})\,\mathrm{curl}'\,\vec{E}'^{(m)}_{sc}(\vec{r}') = (1/i\mu\omega_{inc})\left[\frac{\hat{\rho}'}{\rho'}(-im) - \hat{\phi}'\frac{\partial}{\partial\rho'}\right]\vec{E}'^{(m)}_{sc}(\vec{r}').$$

The special case $\omega = 0$ coincides with Case I.

3.2. Case II: Harmonic Motion

In this mode we consider the special case

$$\vec{v}(t) = G(t)\hat{x}_1,\ G(t) = G\cos(\omega t),\ G = const \tag{3.8}$$

with coordinate transformations

$$x_1 = x_1' + F(t),\ F(t) = (G/\omega)\sin(\omega t),\ x_2 = x_2',\ x_3 = x_3';\ \hat{x}_i = \hat{x'}_i,\ i = 1,2,3. \tag{3.9}$$

Then the incoming electrical field in L-frame reads

$$\begin{aligned}\vec{E}'_{inc}(\vec{r}';t) &= \hat{x}_3'\,e^{ik\hat{n}'_{inc}\cdot\vec{r}'}\,e^{i\Omega\sin(\omega t)}e^{-i\omega_{inc}t} = \hat{x}_3'\sum_{-\infty}^{\infty}J_m(\Omega)e^{ik\rho'\cos(\phi'-\alpha)}e^{-i\omega'^{(m)}_{inc}t}\\ &= \hat{x}_3'\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}J_m(\Omega)J_s(k\rho')\,e^{-is(\phi'-\alpha-\pi/2)}e^{-i\omega'^{(m)}_{inc}t}\\ &= \hat{x}_3'\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}E'^{(m,s)}_{inc}(\vec{r}';t) = \hat{x}_3'\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}E'^{(m,s)}_{inc}(\vec{r}')\,e^{-i\omega'^{(m)}_{inc}t}\end{aligned} \tag{3.10}$$

with $\Omega = (G/\omega)\,k\cos\alpha$, $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega$. The scattered field can be expressed in the form $\vec{E}'_{sc}(\vec{r}';t) = \hat{x}_3'\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}E'^{(m,s)}_{sc}(\vec{r}')\,e^{-i\omega'^{(m)}_{sc}t}$, where each (hypothetical) mode satisfies the boundary value problem

$$\begin{cases}\left(lap' + k^2\right)E'^{(m,s)}_{sc}(\vec{r}') = 0, \quad \text{in medium I}\\ \text{Boundary Condition :}\\ J_m(\Omega)J_s(ka)e^{-is(\phi'-\alpha-\pi/2)}e^{-i\omega'^{(m)}_{inc}t} + E'^{(m,s)}_{sc}(\rho',\phi')\,e^{-i\omega'^{(m)}_{sc}t} = 0, \quad \forall\phi', t\\ \text{Periodicity Condition : } E'^{(m,s)}_{sc}(\rho' = a,\phi') = E'^{(m,s)}_{sc}(\rho' = a,\phi' + 2\pi),\ \forall\phi'\\ \text{Radiation Condition as } \rho' \to \infty\end{cases} \tag{3.11}$$

from which one obtains $\omega'^{(m)}_{sc} = \omega'^{(m)}_{inc}$ and

$$\vec{E}'_{sc}(\rho',\phi';t) = -\hat{x}_3'\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}J_m(\Omega)\frac{J_s(ka)}{H_s^{(1)}(ka)}H_s^{(1)}(k\rho')\,e^{-is(\phi'-\alpha-\pi/2)}e^{-i\omega'^{(m)}_{inc}t} \tag{3.12}$$

(3.13)
$$\vec{E}_{sc}(x_1,x_2;t) = -\hat{x}_3 \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} J_m(\Omega) \frac{J_s(ka)}{H_s^{(1)}(ka)} H_s^{(1)}\left(k\sqrt{(x_1-F(t))^2+x_2^2}\right) \times\ e^{-is\left(\tan^{-1}(x_2/(x_1-F(t)))-\alpha-\pi/2\right)} e^{-i(\omega_{inc}-m\omega)t}$$

3.3. Case III: Rotational Motion

In this case the cylinder is assumed to rotate in counterclockwise direction with uniform angular frequency ω, which obeys the coordinate transformations

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, x_3' = x_3$$

and linear velocity $\vec{v}(t) = \omega a \hat{\phi}(t)$, $\hat{\phi}(t) = -\hat{x}_1 \sin(\omega t) + \hat{x}_2 \cos(\omega t)$. Inserting the polar coordinate maps $\rho = \rho'$, $\phi = \omega t + \phi'$ in (3.1c), the incident field has the L-frame representation

$$\begin{aligned} \vec{E}'_{inc}(\vec{r}';t) &= \hat{x}_3' e^{ik\rho'\cos(\phi'-\alpha+\omega t)} e^{-i\omega_{inc}t} \\ &= \hat{x}_3' \sum_{-\infty}^{\infty} J_m(k\rho')\, e^{-im(\phi'-\alpha-\pi/2+\omega t)} e^{-i\omega_{inc}t} \\ &= \hat{x}_3' \sum_{-\infty}^{\infty} J_m(k\rho')\, e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}'^{(m)}\, t} \\ &= \sum_{-\infty}^{\infty} \vec{E}_{inc}'^{(m)}\ (\vec{r}';t) \end{aligned} \tag{3.14}$$

where we define $\omega_{inc}'^{(m)} = \omega_{inc} + m\omega$. For L-observer, the cylinder is stationary and the ambient medium I is rotating clockwise with linear velocity

$$\vec{v}'(\vec{r}';t) = -\omega\rho'\hat{\phi}'(t), \hat{\phi}'(t) = -\hat{x}_1' \sin(-\omega t) + \hat{x}_2' \cos(-\omega t).$$

That the incident field satisfies the homogeneous frame indifferent wave equation

$$\left(lap' - \frac{1}{c^2}\frac{\diamondsuit'^2}{\diamondsuit' t^2}\right) \vec{E}'_{inc} = \vec{0} \tag{3.15}$$

can be seen upon the substitutions

$$\begin{aligned} lap'\vec{E}'_{inc} &= \hat{x}_3' lap' \left(e^{ik\rho'\cos(\phi'-\alpha+\omega t)}\right) e^{-i\omega_{inc}t} = -k^2 \vec{E}'_{inc}, \\ \vec{v}' \cdot \mathrm{grad}' &= -\omega\frac{\partial}{\partial\phi'}, \frac{\diamondsuit'}{\diamondsuit' t} = \frac{\partial}{\partial t} - \omega\frac{\partial}{\partial\phi'}, \\ \frac{\diamondsuit'}{\diamondsuit' t}\vec{E}'_{inc} &= -i\omega_{inc}\vec{E}'_{inc}, \frac{1}{c^2}\frac{\diamondsuit'^2}{\diamondsuit' t^2}\vec{E}'_{inc} = -\frac{\omega_{inc}^2}{c^2}\vec{E}'_{inc} = -k^2\vec{E}'_{inc} \end{aligned}$$

Then the scattered field can be expressed in the form

$$\vec{E}'_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}_{sc}'^{(m)}(\vec{r}';t) = \hat{x}_3' \sum_{-\infty}^{\infty} E_{sc}'^{(m)}(\vec{r}';t) = \hat{x}_3' \sum_{-\infty}^{\infty} E_{sc}'^{(m)}\ (\vec{r}')\, e^{-i\omega_{sc}'^{(m)}t},$$

where each mode is to be calculated from the boundary value problem
(3.16)

$$\begin{cases} \left(lap' - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + \omega \frac{\partial}{\partial \phi'}\right)^2 \right) E'^{(m)}_{sc}(\vec{r}'; t) = 0, \text{ in medium I} \\ \text{Boundary Condition :} \\ J_m(ka)e^{-im(\phi'-\alpha-\pi/2)}e^{-i\omega'^{(m)}_{inc}t} + E'^{(m)}_{sc}\left(\rho' = a, \phi'\right)e^{-i\omega'^{(m)}_{sc}t} = 0, \ \forall \phi', t \\ \text{Periodicity Condition : } E'^{(m)}_{sc}(\rho', \phi') = E'^{(m)}_{sc}(\rho', \phi' + 2\pi), \ \forall \rho', \phi' \\ \text{Radiation Condition as } \rho' \to \infty \end{cases}$$

We may apply the method of separation of variables proposing

$$E'^{(m)}_{sc}\left(\vec{r}'; t\right) = R^{(m)}(\rho')\Phi\left(\phi'\right)e^{-i\omega'^{(m)}_{inc}t}. \tag{3.17}$$

The component describing angular variation satisfies the reduced boundary value problem

$$\ddot{\Phi}\left(\phi'\right) + v^2\Phi(\phi') = 0, \ \Phi(\phi') = \Phi(\phi' + 2\pi) \tag{3.18}$$

which restricts the separation constant ν to positive integer values $\nu = 1, 2, ...$ for linearly independent eigenmodes, while we choose $\Phi_v(\phi'^{-i\nu(\phi'-\alpha-\pi/2)}$ for convenience. Then the radial component satisfies the Bessel equation

$$\left(\frac{d^2}{d\rho'^2} + \frac{1}{\rho'}\frac{d}{d\rho'} + \left(\frac{\omega'^{(m-\nu)}_{inc}}{c}\right)^2 - \frac{\nu^2}{\rho'^2}\right) R^{(m)}_\nu = 0, \tag{3.19}$$

with $\omega'^{(m-\nu)}_{inc} = \omega_{inc} + (m - \nu)\omega$, which uniquely yields ν-th order Hankel functions of the first kind $H^{(1)}_\nu\left(\frac{\omega'^{(m-\nu)}_{inc}}{c}\rho'\right)$ under the radiation condition. Accordingly, the sought for m-th mode of the scattered field can be written as

$$\vec{E}'^{(m)}_{sc}\left(\vec{r}'; t\right) = \hat{x}'_3 \sum_{\nu=1}^{\infty} a^{(m)}_\nu H^{(1)}_\nu\left(\frac{\omega'^{(m-\nu)}_{inc}}{c}\rho'\right) e^{-i\nu(\phi'-\alpha-\pi/2)}e^{-i\omega'^{(m)}_{inc}t}, \tag{3.20}$$

where the unknown coefficients $a^{(m)}_\nu$ are to be solved from the reduced boundary condition
(3.21)

$$\sum_{\nu=1}^{\infty} a^{(m)}_\nu H^{(1)}_\nu\left(\frac{\omega'^{(m-\nu)}_{inc}}{c}a\right) e^{-i\nu(\phi'-\pi/2)} = -J_m(ka)e^{-im(\phi'-\alpha-\pi/2)}, \ \forall \phi'.$$

Based on the orthogonality property

$$\int_0^{2\pi} e^{+ir(\phi'-\pi/2)}e^{-i\nu(\phi'-\pi/2)}d\phi' = \begin{cases} 0, \nu \neq r \\ 2\pi, \nu = r \end{cases}, \tag{3.22}$$

of the trigonometric functions one can multiply both sides of (3.21) by $e^{+ir(\phi'-\alpha-\pi/2)}$ and integrate w.r.t. ϕ' from 0 to 2π to get

$$\begin{aligned} 2\pi a_r^{(m)} H_r^{(1)}\left(\frac{\omega_{inc}'^{(m-r)}}{c}a\right) &= -J_m(ka)\int_0^{2\pi} e^{+ir(\phi'-\pi/2)}e^{-im(\phi'-\pi/2)}d\phi' \\ &= \begin{cases} 0, m \neq r \\ -2\pi J_m(ka), m = r \end{cases} \end{aligned}$$

which reads

$$a_r^{(m)} = \begin{cases} 0, r \neq m \\ -J_m(ka)/H_m^{(1)}(ka), r = m \end{cases} \tag{3.23}$$

and eventually

$$\vec{E}_{sc}'^{(m)}\ (\vec{r}';t) = -\hat{x}_3' \frac{J_m(ka)}{H_m^{(1)}(ka)} H_m^{(1)}\ (k\rho')\, e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}'^{(m)}t} \tag{3.24}$$

$$\vec{E}_{sc}^{(m)}(\vec{r};t) = -\hat{x}_3 \frac{J_m(ka)}{H_m^{(1)}(ka)} H_m^{(1)}(k\rho) e^{-im(\phi-\alpha-\pi/2)} e^{-i\omega_{inc}t} \tag{3.25}$$

$$\vec{E}_{sc}(\vec{r};t) = -\hat{x}_3 \left[\sum_{-\infty}^{\infty} \frac{J_m(ka)}{H_m^{(1)}(ka)} H_m^{(1)}(k\rho) e^{-im(\phi-\alpha-\pi/2)}\right] e^{-i\omega_{inc}t}. \tag{3.26}$$

It is observed that the total scattered field is monochromatic, having the *same* angular frequency as the incident field and its expression is independent of the angular frequency of rotation ω, coinciding with the result for the stationary case $\omega = 0$.

4. TE Plane Wave Scattering by a Moving Circular Dielectric Cylinder

In this section we shall investigate the scattering of an incident homogeneous monochromatic TE plane wave given in (3.1) by a lossless circular dielectric cylinder with constitutive parameters (ε_d,μ_d) for the same three modes of motion as in Section 3 in a similar fashion. Therefore the same quantities already defined for the corresponding problem in Section 3 will not be repeated.

4.1. Case I: Uniform Motion

As in Section 3.1 the incident, scattered and transmitted fields in L-frame read

$$\begin{aligned} \vec{E}_{inc}'(\vec{r}';t) &= \sum_{-\infty}^{\infty} \vec{E}_{inc}'^{(m)}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}_{inc}'^{(m)}\ (\vec{r}')\, e^{-i\omega_{inc}'t} \\ &= \hat{x}_3' \sum_{-\infty}^{\infty} J_m\ (k\rho')\, e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}'t} \end{aligned} \tag{4.1a}$$

$$\vec{H}'_{inc}(\vec{r}';t) = (1/Z)\,\hat{n}'_{inc} \times \vec{E}'_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{inc}(\vec{r}')e^{-i\omega'_{inc}t} \tag{4.1b}$$

$$\vec{E}'_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}')\,e^{-i\omega'^{(m)}_{sc}t} \tag{4.2a}$$

$$\vec{H}'_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{sc}(\vec{r}')\,e^{-i\omega'^{(m)}_{sc}t} \tag{4.2b}$$

$$\vec{E}'_{d}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{d}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{d}(\vec{r}')\,e^{-i\omega'^{(m)}_{d}t} \tag{4.3a}$$

$$\vec{H}'_{d}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{d}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{d}(\vec{r}')\,e^{-i\omega'^{(m)}_{d}t} \tag{4.3b}$$

which are related by

$$\vec{H}'^{(m)}_{inc}(\vec{r}') = (1/i\mu\omega_{inc})\,\text{curl}'\,\vec{E}'^{(m)}_{inc}(\vec{r}') \tag{4.4a}$$

$$\vec{H}'^{(m)}_{sc}(\vec{r}') = (1/i\mu\omega_{inc})\,\text{curl}'\,\vec{E}'^{(m)}_{sc}(\vec{r}') \tag{4.4b}$$

$$\vec{H}'^{(m)}_{d}(\vec{r}') = (1/i\mu_d\omega_{inc})\,\text{curl}'\,\vec{E}'^{(m)}_{d}(\vec{r}') \tag{4.4c}$$

with $\omega'_{inc} = \omega_{inc}(1 - \beta\cos\alpha)$. These modes satisfy the stationary boundary value problem

$$
(4.5a,k) \quad \begin{cases}
\left(lap' + k^2\right) \begin{pmatrix} \vec{E}'^{(m)}_{sc}(\vec{r}') \\ \vec{H}'^{(m)}_{sc}(\vec{r}') \end{pmatrix} = \vec{0}, \text{in medium I} \\
\left(lap' - \frac{1}{c_d^2}\frac{\partial^2}{\partial t^2}\right) \begin{pmatrix} \vec{E}'^{(m)}_{d}(\vec{r}';t) \\ \vec{H}'^{(m)}_{d}(\vec{r}';t) \end{pmatrix} = \vec{0} \\
or \left(lap' + k_d^2\right) \begin{pmatrix} \vec{E}'^{(m)}_{d}(\vec{r}') \\ \vec{H}'^{(m)}_{d}(\vec{r}') \end{pmatrix} = \vec{0}, \text{in region D} \\
\text{Boundary Conditions :} \\
\vec{E}'^{(m)}_{inc}(\rho' = a, \phi'; t) + \vec{E}'^{(m)}_{sc}(\rho' = a, \phi'; t)\Big|_{\text{tangential}} \\
= \vec{E}'^{(m)}_{d}(\rho' = a, \phi'; t)\Big|_{\text{tangential}}, \forall \phi', t \\
\vec{H}'^{(m)}_{inc}(\rho' = a, \phi'; t) + \vec{H}'^{(m)}_{sc}(\rho' = a, \phi'; t)\Big|_{\text{tangential}} \\
= \vec{H}'^{(m)}_{d}(\rho' = a, \phi'; t)\Big|_{\text{tangential}}, \forall \phi', t \\
\text{Periodicity Conditions :} \\
\vec{E}'^{(m)}_{sc}(\rho', \phi') = \vec{E}'^{(m)}_{sc}(\rho', \phi' + 2\pi), \ \forall \rho', \phi' \\
\vec{E}'^{(m)}_{d}(\rho', \phi') = \vec{E}'^{(m)}_{d}(\rho', \phi' + 2\pi), \ \forall \rho', \phi' \\
\vec{H}'^{(m)}_{sc}(\rho', \phi') = \vec{H}'^{(m)}_{sc}(\rho', \phi' + 2\pi), \ \forall \rho', \phi' \\
\vec{H}'^{(m)}_{d}(\rho', \phi') = \vec{H}'^{(m)}_{d}(\rho', \phi' + 2\pi), \ \forall \rho', \phi' \\
\text{Boundedness Conditions :} \\
\vec{E}'^{(m)}_{d}(\rho' = 0, \phi') = \text{finite}, \forall \phi' \\
\vec{H}'^{(m)}_{d}(\rho' = 0, \phi') = \text{finite}, \ \forall \phi' \\
\text{Radiation Condition for each mode as } \ \rho' \to \infty
\end{cases}
$$

(4.5a,b) uniquely yields

$$
(4.6) \qquad \omega'^{(m)}_{sc} = \omega'_{inc}, \ \omega'^{(m)}_{d} = \omega_{inc}
$$

and the electrical field modes can be represented by

$$
(4.7) \qquad \begin{array}{r} \vec{E}'^{(m)}_{sc}(\rho', \phi') = \hat{x}'_3 A_m H^{(1)}_m(k\rho')\, e^{-im(\phi' - \alpha - \pi/2)}, \\ \vec{E}'^{(m)}_{d}(\rho', \phi') = \hat{x}'_3 B_m J_m(k_d\rho')\, e^{-im(\phi' - \alpha - \pi/2)} \end{array}
$$

The boundary conditions (4.5c,d) require the constants A_m, B_m to be solved from the system

$$
(4.8a) \qquad J_m(ka) + A_m H^{(1)}_m(ka) = B_m J_m(k_d a)
$$

$$
(4.8b) \qquad (k/\mu\omega_{inc}) \left[J'_m(ka) + A_m H'^{(1)}_m(ka)\right] = (k_d/\mu_d\omega_{inc})\, B_m J'_m(k_d a)
$$

to get

$$
(4.9a) \qquad A_m = \frac{1}{\Delta_m}\left[-J'_m(ka)J_m(k_d a) + (Z/Z_d)\, J_m(ka)J'_m(k_d a)\right]
$$

(4.9b) $$B_m = \frac{1}{\Delta_m}\left[-J'_m(ka)H_m^{(1)}(ka) + J_m(ka)H_m^{(1)\prime}(ka)\right] = \frac{(2i/\pi ka)}{\Delta_m}$$

with

(4.9c) $$\Delta_m = J_m(k_d a)H_m^{(1)\prime}(ka) - (Z/Z_d)\,J'_m(k_d a)H_m^{(1)}(ka),$$

while $Z_d = \sqrt{\mu_d/\varepsilon_d}$. Hence, the scattered and transmitted electrical fields in L-frame read

(4.10a) $$\vec{E}'_{sc}(\rho', \phi'; t) = \hat{x}'_3 \sum_{-\infty}^{\infty} A_m H_m^{(1)}(k\rho')\, e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega'_{inc}t}$$

(4.10b) $$\vec{E}'_d(\rho', \phi'; t) = \hat{x}'_3 \sum_{-\infty}^{\infty} B_m J_m(k_d\rho')\, e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}t}$$

(4.11a) $$\begin{aligned}\vec{E}_{sc}(x_1, x_2; t) &= \hat{x}_3 \sum_{-\infty}^{\infty} A_m H_m^{(1)}\left(k\sqrt{(x_1 - Gt)^2 + x_2^2}\right) \\ &\quad \times\, e^{-im\left(\tan^{-1}(x_2/(x_1-Gt))-\alpha-\pi/2\right)} e^{-i\omega_{inc}(1-\beta\cos\alpha)t}\end{aligned}$$

(4.11b) $$\begin{aligned}\vec{E}_d(x_1, x_2; t) &= \hat{x}_3 \sum_{-\infty}^{\infty} B_m J_m\left(k_d\sqrt{(x_1 - Gt)^2 + x_2^2}\right) \\ &\quad \times\, e^{-im\left(\tan^{-1}(x_2/(x_1-Gt))-\alpha-\pi/2\right)} e^{-i\omega_{inc}t}\end{aligned}$$

In the far field of the scatterer one has (3.7a) with the scattering pattern

(4.12) $$F'(\phi') = e^{-i\pi/4}\sqrt{\frac{2}{\pi}} \sum_{-\infty}^{\infty} A_m e^{-im(\phi'-\alpha)}.$$

4.2. Case II: Harmonic Motion

Similar to the case in Section 3.2 the incident, scattered and transmitted fields in L-frame read

(4.13a) $$\vec{E}'_{inc}(\vec{r}'; t) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} \vec{E}'^{(m,s)}_{inc}(\vec{r}')\, e^{-i\omega'^{(m)}_{inc}t}$$

(4.13b) $$\vec{H}'_{inc}(\vec{r}'; t) = (1/Z)\,\hat{n}'_{inc} \times \vec{E}'_{inc}(\vec{r}'; t) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} \vec{H}'^{(m,s)}_{inc}(\vec{r}')\, e^{-i\omega'^{(m)}_{inc}t}$$

(4.14) $$\begin{aligned}\vec{E}'_{sc}(\vec{r}'; t) &= \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} \vec{E}'^{(m,s)}_{sc}(\vec{r}')\, e^{-i\omega'^{(m)}_{inc}t},\ \vec{H}'_{sc}(\vec{r}'; t) \\ &= \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} \vec{H}'^{(m,s)}_{sc}(\vec{r}')\, e^{-i\omega'^{(m)}_{inc}t}\end{aligned}$$

(4.15)
$$\vec{E}'_d(\vec{r}';t) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} \vec{E}'^{(m,s)}_d(\vec{r}')\, e^{-i\omega_{inc}t},\ \vec{H}'_d(\vec{r}';t) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} \vec{H}'^{(m,s)}_d(\vec{r}')\, e^{-i\omega_{inc}t}$$

with

(4.16a)
$$\vec{E}'^{(m,s)}_{inc}(\vec{r}') = \hat{x}'_3 J_m(\Omega) J_s(k\rho')\, e^{-is(\phi'-\alpha-\pi/2)}$$

(4.16b)
$$\vec{E}'^{(m,s)}_{sc}(\vec{r}') = \hat{x}'_3 A_{m,s} J_m(\Omega) H^{(1)}_s(k\rho')\, e^{-is(\phi'-\alpha-\pi/2)}$$

(4.16c)
$$\vec{E}'^{(m,s)}_{d}(\vec{r}') = \hat{x}'_3 J_m(\Omega) B_{m,s} J_s(k_d\rho')\, e^{-is(\phi'-\alpha-\pi/2)}$$

and

(4.17a)
$$\vec{H}'^{(m,s)}_{inc}(\vec{r}') = (1/i\mu\omega_{inc})\,\mathrm{curl}'\,\vec{E}'^{(m,s)}_{inc}(\vec{r}')$$

(4.17b)
$$\vec{H}'^{(m,s)}_{sc}(\vec{r}') = (1/i\mu\omega_{inc})\,\mathrm{curl}'\,\vec{E}'^{(m,s)}_{sc}(\vec{r}')$$

(4.17c)
$$\vec{H}'^{(m,s)}_{d}(\vec{r}') = (1/i\mu_d\omega_{inc})\,\mathrm{curl}'\,\vec{E}'^{(m,s)}_{d}(\vec{r}')$$

while $\Omega = (G/\omega)\,k\cos\alpha$, $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega$. The boundary conditions require the constants $A_{m,s}$, $B_{m,s}$ to be solved from the system

(4.18a)
$$\left[J_s(ka) + A_{m,s}H^{(1)}_s(ka)\right] e^{-i\omega'^{(m)}_{inc}t} = B_{m,s}J_s(k_da)e^{-i\omega_{inc}t}$$

(4.18b)
$$(k/\mu\omega_{inc})\left[J'_s(ka) + A_{m,s}H^{(1)'}_s(ka)\right] e^{-i\omega'^{(m)}_{inc}t} = (k_d/\mu_d\omega_{inc})\,B_{m,s}J'_s(k_da)e^{-i\omega_{inc}t}$$

to get

(4.19a)
$$A_{m,s} = \frac{1}{\Delta_{m,s}}\left[-J'_s(ka)J_s(k_da) + (Z/Z_d)\,J_s(ka)J'_s(k_da)\right]$$

(4.19b)
$$B_{m,s} = \frac{1}{\Delta_{m,s}}\left[-J'_s(ka)H^{(1)}_s(ka) + J_s(ka)H^{(1)'}_s(ka)\right] e^{im\omega t} = \frac{(2i/\pi ka)}{\Delta_{m,s}}e^{im\omega t}$$

with

(4.19c)
$$\Delta_{m,s} = J_s(k_da)H^{(1)'}_s(ka) - (Z/Z_d)\,J'_s(k_da)H^{(1)}_s(ka)$$

Hence, the scattered and transmitted electrical fields in L-frame read

(4.20a)
$$\vec{E}'_{sc}(\rho',\phi';t) = \hat{x}'_3\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} A_{m,s}J_m(\Omega)H^{(1)}_s(k\rho')\, e^{-is(\phi'-\alpha-\pi/2)}\, e^{-i\omega'^{(m)}_{inc}t}$$

(4.20b)

$$\vec{E}_d'(\rho',\phi';t)=\hat{x}_3'\,(2i/\pi ka)\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}\frac{1}{\Delta_{m,s}}J_m(\Omega)J_s\,(k_d\rho')\,e^{-is(\phi'-\alpha-\pi/2)}\,e^{-i\omega_{inc}'^{(m)}t}$$

(4.21a)

$$\vec{E}_{sc}(x_1,x_2;t)=\hat{x}_3\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}A_{m,s}J_m(\Omega)H_s^{(1)}\left(k\sqrt{(x_1-F(t))^2+x_2^2}\right)\\ \times\; e^{-is\left(\tan^{-1}(x_2/(x_1-F(t)))-\alpha-\pi/2\right)}\,e^{-i(\omega_{inc}-m\omega)t}$$

(4.21b)

$$\vec{E}_d(x_1,x_2;t)=\hat{x}_3\,(2i/\pi ka)\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}\tfrac{1}{\Delta_{m,s}}J_m(\Omega)J_s\left(k_d\sqrt{(x_1-F(t))^2+x_2^2}\right)\\ \times\, e^{-is\left(\tan^{-1}(x_2/(x_1-F(t)))-\alpha-\pi/2\right)}\,e^{-i(\omega_{inc}-m\omega)t}$$

4.3. Case III: Rotational Motion

Similar to the case in Section 3.3 the incident fields in L-frame read

(4.22)

$$\vec{E}_{inc}'(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{E}_{inc}'^{(m)}(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{E}_{inc}'^{(m)}(\vec{r}')\,e^{-i\omega_{inc}'^{(m)}t},\\ \vec{E}_{inc}'^{(m)}(\vec{r}')=\hat{x}_3'J_m\,(k\rho')\,e^{-im(\phi'-\alpha-\pi/2)}$$

(4.23)

$$\vec{H}_{inc}'(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{H}_{inc}'^{(m)}(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{H}_{inc}'^{(m)}(\vec{r}')\,e^{-i\omega_{inc}'^{(m)}t},\\ \vec{H}_{inc}'^{(m)}(\vec{r}')=(1/i\mu\omega_{inc})\,\mathrm{curl}'\,\vec{E}_{inc}'^{(m)}(\vec{r}')$$

with $\omega_{inc}'^{(m)}=\omega_{inc}+m\omega$. Then the scattered and transmitted fields can be considered in the form

$$\vec{E}_{sc}'(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{E}_{sc}'^{(m)}(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{E}_{sc}'^{(m)}(\vec{r}')\,e^{-i\omega_{sc}'^{(m)}t},\\ \vec{H}_{sc}'(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{H}_{sc}'^{(m)}(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{H}_{sc}'^{(m)}(\vec{r}')\,e^{-i\omega_{sc}'^{(m)}t}$$

$$\vec{E}_d'(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{E}_d'^{(m)}(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{E}_d'^{(m)}(\vec{r}')\,e^{-i\omega_d'^{(m)}t},\\ \vec{H}_d'(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{H}_d'^{(m)}(\vec{r}';t)=\sum_{-\infty}^{\infty}\vec{H}_d'^{(m)}(\vec{r}')\,e^{-i\omega_d'^{(m)}t}$$

These modes satisfy the stationary boundary value problem

$$
(4.24)\quad
\begin{cases}
\left(lap' - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + \omega\frac{\partial}{\partial \phi'}\right)^2\right)\begin{pmatrix}\vec{E}'^{(m)}_{sc}(\vec{r}';t)\\ \vec{H}'^{(m)}_{sc}(\vec{r}';t)\end{pmatrix} = \vec{0} \text{ or}\\
\left(lap' + k^2\right)\begin{pmatrix}\vec{E}'^{(m)}_{sc}(\vec{r}')\\ \vec{H}'^{(m)}_{sc}(\vec{r}')\end{pmatrix} = \vec{0}, \text{in medium I}\\
\left(lap' - \frac{1}{c_d^2}\frac{\partial^2}{\partial t^2}\right)\begin{pmatrix}\vec{E}'^{(m)}_{d}(\vec{r}';t)\\ \vec{H}'^{(m)}_{d}(\vec{r}';t)\end{pmatrix} = \vec{0} \text{ or}\\
\left(lap' + k_d^2\right)\begin{pmatrix}\vec{E}'^{(m)}_{d}(\vec{r}')\\ \vec{H}'^{(m)}_{d}(\vec{r}')\end{pmatrix} = \vec{0}, \text{in region D}\\
\text{Boundary Conditions :}\\
\vec{E}'^{(m)}_{inc}(\rho' = a, \phi'; t) + \vec{E}'^{(m)}_{sc}(\rho' = a, \phi'; t)\Big|_{\text{tangential}}\\
= \vec{E}'^{(m)}_{d}(\rho' = a, \phi'; t)\Big|_{\text{tangential}}, \ \forall \phi', t\\
\vec{H}'^{(m)}_{inc}(\rho' = a, \phi'; t) + \vec{H}'^{(m)}_{sc}(\rho' = a, \phi'; t)\Big|_{\text{tangential}}\\
= \vec{H}'^{(m)}_{d}(\rho' = a, \phi'; t)\Big|_{\text{tangential}}, \ \forall \phi', t\\
\text{Periodicity Conditions :}\\
\vec{E}'^{(m)}_{sc}(\rho', \phi') = \vec{E}'^{(m)}_{sc}(\rho', \phi' + 2\pi), \ \forall \rho', \phi'\\
\vec{E}'^{(m)}_{d}(\rho', \phi') = \vec{E}'^{(m)}_{d}(\rho', \phi' + 2\pi), \ \forall \rho', \phi'\\
\vec{H}'^{(m)}_{sc}(\rho', \phi') = \vec{H}'^{(m)}_{sc}(\rho', \phi' + 2\pi), \ \forall \rho', \phi'\\
\vec{H}'^{(m)}_{d}(\rho', \phi') = \vec{H}'^{(m)}_{d}(\rho', \phi' + 2\pi), \ \forall \rho', \phi'\\
\text{Boundedness Conditions :}\\
\vec{E}'^{(m)}_{d}(\rho' = 0, \phi') = \text{finite}, \ \forall \phi'\\
\vec{H}'^{(m)}_{d}(\rho' = 0, \phi') = \text{finite}, \ \forall \phi'\\
\text{Radiation Condition for each mode as } \ \rho' \to \infty
\end{cases}
$$

which directly yield $\omega'^{(m)}_{sc} = \omega'^{(m)}_{inc}$, $\omega'^{(m)}_{d} = \omega_{inc}$ and

$$
(4.25a)\quad \vec{E}'^{(m)}_{sc}\ (\vec{r}'; t) = \hat{x}'_3 \sum_{\nu=1}^{\infty} a^{(m)}_{\nu} H^{(1)}_{\nu}\left(\frac{\omega'^{(m-\nu)}_{inc}}{c}\rho'\right) e^{-i\nu(\phi' - \alpha - \pi/2)} e^{-i\omega'^{(m)}_{inc} t}
$$

$$
(4.25b)\quad \vec{E}'^{(m)}_{d}\ (\vec{r}'; t) = \hat{x}'_3 \sum_{\nu=1}^{\infty} b^{(m)}_{\nu} J_v\left(k_d \rho'\right) e^{-i\nu(\phi' - \alpha - \pi/2)} e^{-i\omega_{inc} t}
$$

with $\omega'^{(m-\nu)}_{inc} = \omega_{inc} + (m - \nu)\omega$, while

$$
(4.26)\quad \vec{H}'^{(m)}_{sc}(\vec{r}') = (1/i\mu\omega_{inc}) \operatorname{curl}' \vec{E}'^{(m)}_{sc}(\vec{r}'), \vec{H}'^{(m)}_{d}(\vec{r}') = (1/i\mu_d\omega_{inc}) \operatorname{curl}' \vec{E}'^{(m)}_{d}(\vec{r}').
$$

The unknown coefficients $a^{(m)}_{\nu}$, $b^{(m)}_{\nu}$ are to be solved from the reduced boundary conditions

(4.27a)

$$\left[J_m(ka)e^{-im(\phi'-\alpha-\pi/2)} + \sum_{\nu=1}^{\infty} a_\nu^{(m)} H_\nu^{(1)}\left(\frac{\omega_{inc}^{\prime(m-\nu)}}{c}a\right) e^{-i\nu(\phi'-\pi/2)}\right] e^{-i\omega_{inc}^{\prime(m)}t} = \\ = \sum_{\nu=1}^{\infty} b_\nu^{(m)} J_v\left(k_d a\right) e^{-i\nu(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}t}$$

(4.27b)

$$\left[kJ_m'(ka)e^{-im(\phi'-\alpha-\pi/2)} + \sum_{\nu=1}^{\infty} a_\nu^{(m)} \frac{\omega_{inc}^{\prime(m-\nu)}}{c} H_\nu^{(1)\prime}\left(\frac{\omega_{inc}^{\prime(m-\nu)}}{c}a\right) e^{-i\nu(\phi'-\pi/2)}\right] \\ \times \frac{e^{-i\omega_{inc}^{\prime(m)}t}}{\mu\omega_{inc}} = \frac{k_d}{\mu_d\omega_{inc}} \sum_{\nu=1}^{\infty} b_\nu^{(m)} J_v'\left(k_d a\right) e^{-i\nu(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}t}$$

for $\forall\phi', t$. Based on the orthogonality property (3.22), one can multiply both sides of (4.27) by $e^{+ir(\phi'-\alpha-\pi/2)}$ and integrate w.r.t. ϕ' from 0 to 2π to get $a_r^{(m)}, b_r^{(m)} = 0$, $r \neq m$, while

(4.28a) $$a_r^{(r)} = \frac{1}{\Delta_r}\left[-J_r'(ka)J_r(k_d a) + (Z/Z_d)\, J_r(ka)J_r'(k_d a)\right] \triangleq A_r$$

(4.28b)

$$b_r^{(r)} = \frac{1}{\Delta_r}\left[-J_r(ka)H_r^{(1)}(ka) + J_r(ka)H_r^{(1)\prime}(ka)\right] e^{-ir\omega t} = \frac{(2i/\pi ka)}{\Delta_r} e^{-ir\omega t}$$

with

(4.28c) $$\Delta_r = J_r(k_d a)H_r^{(1)\prime}(ka) - (Z/Z_d)\, J_r'(k_d a)H_r^{(1)}(ka),$$

which read

(4.29a) $$\vec{E}_{sc}^{\prime(m)}\ (\vec{r}\,';t) = \hat{x}_3' A_m H_m^{(1)}\left(k\rho'\right) e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}^{\prime(m)}t}$$

(4.29b) $$\vec{E}_d^{\prime(m)}\ (\vec{r}\,';t) = \hat{x}_3' \left(2i/\pi ka\right) \frac{1}{\Delta_m} J_m\left(k_d\rho'\right) e^{-im(\phi'-\alpha-\pi/2)} e^{-i\omega_{inc}^{\prime(m)}t}$$

(4.30a) $$\vec{E}_{sc}^{(m)}(\vec{r};t) = \hat{x}_3 A_m H_m^{(1)}(k\rho) e^{-im(\phi-\alpha-\pi/2)} e^{-i\omega_{inc}t}$$

(4.30b) $$\vec{E}_d^{(m)}(\vec{r};t) = \hat{x}_3 \left(2i/\pi ka\right) \frac{1}{\Delta_m} J_m(k_d\rho) e^{-im(\phi-\alpha-\pi/2)} e^{-i\omega_{inc}t}$$

(4.31a) $$\vec{E}_{sc}(\vec{r};t) = \hat{x}_3 \left[\sum_{-\infty}^{\infty} A_m H_m^{(1)}(k\rho) e^{-im(\phi-\alpha-\pi/2)}\right] e^{-i\omega_{inc}t}$$

(4.31b) $$\vec{E}_d(\vec{r};t) = \hat{x}_3 \left[(2i/\pi ka) \sum_{-\infty}^{\infty} \frac{1}{\Delta_m} J_m(k_d\rho) e^{-im(\phi-\alpha-\pi/2)}\right] e^{-i\omega_{inc}t}$$

Similar to the case with PEC cylinder in Section 3.3, it is observed that the total scattered and transmitted fields (4.31) are monochromatic, having the *same* angular frequency as the incident field. Their expressions are independent of the angular frequency of rotation ω, coinciding with the result for the stationary case $\omega = 0$.

5. Concluding Remarks

With the wide acceptance of Special and General Relativity Theories (SRT&GRT) as experimental facts in early 20^{th} century, problems of electromagnetic wave propagation and scattering for moving bodies have been handled and solved mostly in a relativistic frame in literature till date in the context of the principle of special or general space-time covariance. With relevance to the cases investigated in the present work, the solution of the problem of plane wave scattering by PEC and dielectric circular cylinders in uniform motion can be seen at Section 5.16 of [2] and the references cited therein. For a comparison in this mode it should be realized that the Euclidean transformations of HE for rigid bodies reduce to the Galilean transformations

$$x_1' = x_1 - Gt,\ t' = t \tag{5.1a}$$

while SRT assumes the standard Lorentz transformations

$$x_1' = \gamma\left(x_1 - Gt\right),\ t' = \gamma\left(t - \beta x_1/c\right) \tag{5.1b}$$

with $\gamma = 1/\sqrt{1-\beta^2}$. A first order approximation in β reads first order Lorentz transformations

$$x_1' = x_1 - Gt,\ t' = t - \beta x_1/c, \tag{5.1c}$$

which also indicates that a first or higher order departure in β should be expected between the physical quantities to be calculated with HE and SRT. For bodies in nonuniform motion SRT no more applies since Lorentz transformations cannot be generalized directly for nonuniform velocities while keeping the Maxwell equations form invariant. In that context one can mention to the heuristic approaches in the works of Censor (see [3] and the references cited therein) where a number of canonical scattering problems including oscillating circular cylinders are investigated. Regarding plane wave scattering by a circular cylinder in rotational motion, we observe the same phenomenon in both frame indifferent and form invariant formulations for PEC case, while for a dielectric cylinder the scattering coefficients are dependent on the rotation frequency in relativity theory (cf.[2], Sections 10.7,10.8), unlike the prediction of HE.

With the scope of reviving interest in HE, the present work is planned to be extended to demonstrate the predictions of HE for a broad set of canonical problems with important applications. In that context the problems of plane wave scattering by a moving PEC plane and a dielectric half-space in uniform and harmonic motions have also been investigated by the present author elsewhere [4].

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