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Serkan Doganalp
Konya Technical University

Cevat Inal
Selcuk University

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Res. Assist. S. Doğanalp[1], Assoc. Prof. Dr. B. Turgut[1], Prof. Dr. C. İnal[2]

[1] Selçuk University, Faculty of Engineering and Architecture, Department of Geomatic Engineering Division of Geodesy, **Konya/TURKEY**
[2] Selçuk University, Faculty of Engineering and Architecture, Department of Geomatic Engineering Division of Surveying Technique, **Konya/TURKEY**

ABSTRACT

Deformation measurements and analysis are one of the most significant subjects in geodesy. Deformation monitoring and taking precautions on time is very important for safety of people and facilities.

In this research, three period measurements were done in a trigonometric levelling network. S transformation and Kalman filtering techniques were performed for detection of vertical deformations. Results showed that both methods determined vertical displacements at the same points. Moreover, the values of velocity and acceleration of points were calculated using the Kalman filtering technique.

Keywords: Deformation Analysis, Kalman Filtering Technique, S Transformation

INTRODUCTION

Some deformation models have been developed such as quasi-static, static, kinematics and dynamic. The most general model is the dynamic model. In dynamic model, the geometric changes related to the monitored object are taken into account as a function of time and function of efforts impacting the object. In kinematics model, the velocity vectors with geometric changes, in static model the efforts impacting the object with geometric changes and in quasi-static model only the geometric changes are considered.

θ^2 – criteria or S transformation technique are generally used for determining quasi-static deformations, on the other hand Kalman filtering technique is preferred for kinematic model. The mathematical model of deformation analysis with S transformation is more appropriate than other methods in the context of programming technique. There is a frequent datum change in deformation analysis. S transformation provided the transition from one datum to another. Station coordinates can therefore be computed in any datum without need for a new adjustment. Kinematic model have to be scrutinized in addition to quasi-static model. In this respect, the Kalman filtering technique can be widely used to determine the changes of position, velocity and acceleration.

DEFORMATION ANALYSIS WITH S TRANSFORMATION

The period measurements are adjusted separately by free network adjustment method in the analysis of the deformation measurements. The reason of choosing free network adjustment method is to avoid possible measurement errors. In first step, free adjustment method is performed in the deformation analysis. In second step, the outlier measurements are determined, and finally deformations can be determined by performing global test for detecting whether the network has deformation or not.

To interpret deformation analysis in a geodetic network, all period measurements must be used in the same datum. The S transformation technique was first applied in 1950 by Baarda for monitoring of point displacements without using another adjustment method. S-transformation matrix can be calculated as follows:

$$S = I - G\left(B^T G\right)^{-1} B^T \qquad (B = E.G) \tag{1}$$

where I is the unit matrix, E is the datum selector matrix (a diagonal matrix including 1 or 0) and G is the matrix of eigenvectors corresponding to defect number of eigenvalues of matrix N which is the normal equation coefficients matrix. The adjustment results in any i datum is transformed into j datum with S-transformation by using equations (2), (3), (4) and (5) [1], [4], [7], [12], [16].

$$G = \begin{bmatrix} G_e \\ G_b \end{bmatrix} \quad \text{and} \quad B_j = E_j G = \begin{bmatrix} G_e \\ 0 \end{bmatrix} \tag{2}$$

$$S_j = I - G\left(B_j^T G\right)^{-1} B_j^T \tag{3}$$

$$X_j = S_j X_i \tag{4}$$

$$Q_{xx}^j = S_j Q_{xx}^i S_j^T \tag{5}$$

In a network measured in time t_n, the X_i parameter vector related to i datum and the weight coefficients matrix are;

$$X_i = \begin{bmatrix} X_e^i \\ X_b^i \end{bmatrix} \tag{6}$$

$$Q_{xx}^i = \begin{bmatrix} Q_{ee}^i & Q_{eb}^i \\ Q_{be}^i & Q_{bb}^i \end{bmatrix} \tag{7}$$

where "*e*" is conjugate (datum) points which are determined by free adjustment method having their coordinates, the coordinates of other points defined as "*b*" and the other unknowns. Transformation process separately is performed for the 1st period and 2nd period. $\left(Q_{ee}^j\right)_1$ and $\left(Q_{ee}^j\right)_2$ are weight coefficients matrix, $\left(X_e^j\right)_1$ and $\left(X_e^j\right)_2$ are coordinate unknowns in j datum. The global test for congruency points is calculated with;

$$H_0 : E\left(X_e^j\right)_1 = E\left(X_e^j\right)_2 \tag{8}$$

$$d_e = \left(X_e^j\right)_2 - \left(X_e^j\right)_1 \tag{9}$$

$$\left(Q_{dd}\right)_e = \left(Q_{ee}^j\right)_1 + \left(Q_{ee}^j\right)_2 \tag{10}$$

$$R_e = d_e^T \left(Q_{dd}\right)_e^+ d_e \tag{11}$$

$$F = \frac{R_e}{m_0^2 h_e} \tag{12}$$

where F is test value, $h_e = u_e - d$ is degrees of freedom of R_e. $F > F_{h_e, f, 1-\alpha}$ is judged to be a deformation at the section formed by the conjugate points of the network. The common variance m^2 that will be valid for 1st and 2nd periods is calculated [4], [11], [12]. S transformation technique for the detailed information can be obtained from [1], [4], [6], [7], [11], [12], [16]. After this stage, significant point movements can be determined by using S transformation. Kalman filtering technique can be performed after this process by using these determined moving points.

KALMAN FILTERING TECHNIQUE

Kalman filtering technique was introduced in 1960 by Rudolf Emil Kalman and has been used in engineering practices and many areas in later [13], [14], [15]. Kalman filtering technique is composed of three basic stages such as prediction, filtering and smoothing. The Kalman filtering technique is used for the estimation of time-dependent unknown parameters by the least squares principle [3], [5]. A time-dependent 1D kinematic model that contains height, velocity and acceleration can be expressed by the following formulae:

$$H_j^{(i)} = H_j^{(i-1)} + (t_i - t_{i-1})v_j^{(i-1)} + \frac{1}{2}(t_i - t_{i-1})^2 a_j^{(i-1)} \qquad (13)$$

where

$H_j^{(i)}$	height of point *j* at time *(i)* period
$H_j^{(i-1)}$	height of point *j* at time *(i-1)* period
v_j	velocity of height of *j* point
a_j	acceleration of height of *j* point
i	1,2,…n (n: measurement period number)
j	1,2,…m (m: number of points)

The velocity and acceleration are the first and the second derivations of the position with respect to time. Equation (13) is expanded with the velocity and accelerations. As a result of this expansion the following equation is obtained.

$$\begin{aligned} H_j^{(i)} &= H_j^{(i-1)} + (t_i - t_{i-1})v_j^{(i-1)} + \frac{1}{2}(t_i - t_{i-1})^2 a_j^{(i-1)} \\ v_j^{(i)} &= v_j^{(i-1)} + (t_i - t_{i-1})a_j^{(i-1)} \qquad (14) \\ a_j^{(i)} &= a_j^{(i-1)} \end{aligned}$$

Equation (14) can be summarized and expressed by matrix form as follows.

$$\overline{Y}_i = \begin{bmatrix} H_j \\ v_j \\ a_j \end{bmatrix}_i = \begin{bmatrix} I & I(t_i - t_{i-1}) & I\frac{(t_i - t_{i-1})^2}{2} \\ 0 & I & I(t_i - t_{i-1}) \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} H_j \\ v_j \\ a_j \end{bmatrix}_{i-1} \qquad (15)$$

$$\overline{Y}_i = T_{i,i-1}\hat{Y}_{i-1} \qquad (16)$$

where

$\overline{Y}_i$	prediction status (height, velocity, acceleration) vector at time t_i
$\hat{Y}_{i-1}$	state (height, velocity, acceleration) vector at time t_{i-1}
$T_{i,i-1}$	transition matrix from t_{i-1} to t_i
I	unit matrix

Equation (16) is the basic prediction equation of Kalman filtering technique. So that the prediction equation and the covariance matrix is as follows [8], [18], [19].

$$\overline{Y}_i = T_{i,i-1}\hat{Y}_{i-1} + S_{i,i-1} + w_{i-1} \qquad (17)$$

$$Q_{\overline{Y}\overline{Y},i} = T_{i,i-1}\, Q_{\hat{Y}\hat{Y},i-1}\, T_{i,i-1}^{\ T} + S_{i,i-1}\, Q_{ww,i-1}\, S_{i,i-1}^{\ \mathrm{T}} \tag{18}$$

$$S_{i,i-1}^{\ T} = \begin{bmatrix} I\dfrac{(t_i - t_{i-1})^2}{2} & I(t_i - t_{i-1}) & I \end{bmatrix} \tag{19}$$

where

w_{i-1}	the random noise vector between t_i and t_{i-1}
$Q_{\hat{Y}\hat{Y},i-1}$	the cofactor matrix of state vector at time t_{i-1}
$Q_{ww,i-1}$	the cofactor matrix of system noises at time t_{i-1}
$S_{i,i-1}^{\ T}$	noise matrix that consist of the terms of the last column $T_{i,i-1}$
I	unit matrix

The random noise vector w is uncertain and, as a rule can not be measured. For this reason, pseudo observation vector can be used as $w = 0$. The effect of noise on the positions can be determined using the former experiences. According to [10] and [17], this effect on velocity and acceleration are difficult to be predicted. The cofactor matrix of system noises at time t_{i-1} may be derived from the following equation [2], [9].

$$Q_{ww,i-1} = 4(t_i - t_{i-1})^{-4} Q_{SS,i-1} \tag{20}$$

where Q_{SS} is the covariance matrix of the points in time *(i-1)* period. The residual vector is formed in period (i) as follows:

$$l_i + v_{l,i} = A_i \hat{Y}_i = \begin{bmatrix} A_{\mathrm{height},i} & 0 & 0 \end{bmatrix} \begin{bmatrix} Y \\ \dot{Y} \\ \ddot{Y} \end{bmatrix}_{\mathrm{i}} \tag{21}$$

Functional and stochastic models of Kalman filtering technique can be obtained by combining equation (17) and equation (21) as follows [1], [19].

$$\begin{bmatrix} \overline{Y}_i \\ l_i \end{bmatrix} = \begin{bmatrix} I \\ A_i \end{bmatrix} \hat{Y}_i - \begin{bmatrix} v_{\overline{Y},i} \\ v_{l,i} \end{bmatrix} \text{ and } Q_i = \begin{bmatrix} Q_{\overline{Y}\overline{Y},i} & 0 \\ 0 & Q_{ll,i} \end{bmatrix} \tag{22}$$

Kalman gain matrix (K_i) is given as;

$$K_i = Q_{\overline{Y}\overline{Y},i} A_i^{\ T} (Q_{ll,i} + A_i Q_{\overline{Y}\overline{Y},i} A_i^{\ T})^{-1} = Q_{\overline{Y}\overline{Y},i} A_i^{\ T} D_i^{-1} \tag{23}$$

Using the equations above, innovation vector d_i, state vector filtered at time t_i; $\hat{Y}_i$, predicted state vector; $v_{\overline{Y},i}$, residual vector of observations at time t_i can be computed by the following equation:

$$\begin{bmatrix} d_i \\ \hat{Y}_i \\ v_{\overline{Y},i} \\ v_{l,i} \end{bmatrix} = \begin{bmatrix} -A_i & I \\ I - K_i A_l & K_i \\ -K_i A_l & K_i \\ Q_{ll,i} D_i^{-1} A_l & -Q_{ll,i} D_i^{-1} \end{bmatrix} \begin{bmatrix} \overline{Y}_i \\ l_i \end{bmatrix} \tag{24}$$

Actually, the filtering phase is based on classical least squares adjustment. The most important difference between the filtering phase and the classical adjustment procedure is that, contrary to the classical approach, in the filtering the number of observations can be less than the number of unknowns. Through the filtering, adjusted values of state unknowns are computed using weighted combination of measurements and a priori estimations [1], [2], [8].

Test of significance of motion parameters

The calculated parameters (height, velocity and acceleration) determined by Kalman filtering technique should be tested whether significant or not. The test values for the test of unknowns (height, velocity and acceleration) for each point can be calculated as follows [20].

$$T_{h_i} = \frac{|h_i|}{m_{h_i}} \quad , \quad T_{\dot{h}_i} = \frac{|\dot{h}_i|}{m_{\dot{h}_i}} \quad , \quad T_{\ddot{h}_i} = \frac{|\ddot{h}_i|}{m_{\ddot{h}_i}} \tag{25}$$

$T_{h_i} \geq t - Table$, $T_{\dot{h}_i} \geq t - Table$, $T_{\ddot{h}_i} \geq t - Table$

where $T_{h_i}, T_{\dot{h}_i}, T_{\ddot{h}_i}$ are the test values for position, velocity and acceleration respectively and are significant in the case of they are greater than $t - Table$ value.

NUMERICAL APPLICATION

Deformation network is a leveling network which consists of 5 points (Figure 1). Measurements have been performed as 3 periods between January 2004 – June 2004 – December 2004. At first, adjusted heights, variance-covariance matrixes and variance of unit weights of points calculated for each period by free adjustment method separately. The free network adjustment procedure and outlier detection have been computed by Mittermayer and Pope method respectively. Results of free-network adjusment are illustrated in Table 1. After this process in order to determine the moving points the deformation analysis by S transformation technique have been used. Table 2 illustrates calculated results of static deformation analysis (S transformation).

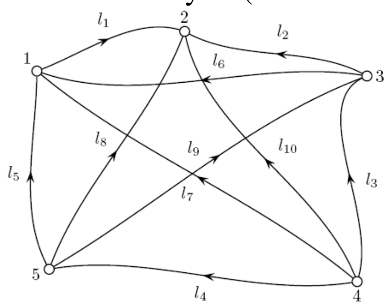

Figure 1. Deformation network

Table 1. Results of free-network adjustment

Periods	t_o (January 04)		t_1 (June 04)		t_2 (December 04)	
m_0 (mm)	0.465448		0.535873		0.431581	
Point Number	Adjusted measures(m)	m_x (mm)	Adjusted measures(m)	m_x (mm)	Adjusted measures(m)	m_x (mm)
1	99.9980	0.6	99.9956	0.6	99.9959	0.5
2	103.8267	0.6	103.8212	0.6	103.8224	0.5
3	90.3302	0.6	90.3310	0.7	90.3318	0.5
4	80.5613	0.5	80.5642	0.6	80.5620	0.5
5	81.2466	0.5	81.2510	0.6	81.2509	0.5

Table 2. Results of static deformation analysis (S transformation)

Periods	t_0 - t_1	t_0 - t_2
* Homogeneity test and common variance account	F_h = 1.326 F_t = 4.284 m_{ort} = 0.502 mm	F_h = 1.163 F_t = 4.284 m_{ort} = 0.449 mm
** Global test	F = 16.484 F_t = 3.259	F = 15.155 F_t = 3.259
Moving points	2,1	5,2,1

* If $F_h < F_t$ then variances are homogeneity
** If $F > F_t$ then in network are significant deformations.

Table 3. Results of statistical test of kinematic model

Global Test		Expanded Model Test	
s_0	0.4654	s_0	0.4536
m_0	0.4536	m_g	0.1964
T	1.0528	T_g	82.6885
q	3.2172	q	3.3258

Decision=Height + Velocity + Acceleration

Results of statistical test of kinematic model are illustrated in Table 3. Where a priori variance (s_0) is computed preliminary adjustment, a posteriori variance (m_0) is computed from the model, T is test value and q is F-distribution table value (confidence level is 0.05). Test value (T) is compared with the F-distribution table value (q) and if T<q then global test is valid. A posteriori variance of expanded model (m_g) is computed from the parameters of the expanded model. T_g is test value for expanded model, q is F-distribution table value for expanded model (confidence level is 0.05). Test value (T_g) is compared with the F-distribution table value (q) and if T_g >q then model is expanded. Results of the test values for height, velocity and acceleration are illustrated in Table 4. Table 2 and Table 4 show that the results for both models are generally similar for point movements. When Table 2 and Table 4 are examined, it can be seen that the obtained results in both models are same. As a result, it can be said that both model results are harmonious. Table 5 illustrates calculated results of height, velocity and acceleration of the network points determined with static and kinematic model. Figure 2 shows the displacement values determined with static and kinematic model in graphical view.

Table 4. The test values for height, velocity and acceleration (Kalman Filter)

PN	Height	Significant	Velocity	Significant	Acceleration	Significant
1	3.3740	[+]	2.1324	[+]	2.5855	[+]
2	3.7844	[+]	1.5583	[-]	1.4398	[-]
3	1.5021	[-]	0.8770	[-]	0.6629	[-]
4	1.0902	[-]	1.1077	[-]	0.6215	[-]
5	5.0610	[+]	0.5495	[-]	0.3304	[-]

q = 2.1314, [-] = insignificant, [+] = significant

Table 5. Height, velocity and acceleration of the network points determined with static and kinematic model

January – June – December 2004				
	Static Model	**Kinematic Model**		
Point Number	Height (mm)	Height (mm)	Velocity (mm / month)	Acceleration (mm / month^2)
1	-2.1213	-4.0996	0.6873	0.1089
2	-4.3130	-4.5976	0.5036	0.0610
3	1.5622	1.6257	0.1365	0.0098
4	0.6459	1.1558	-0.1650	-0.0085
5	4.2261	5.8839	0.1288	0.0081

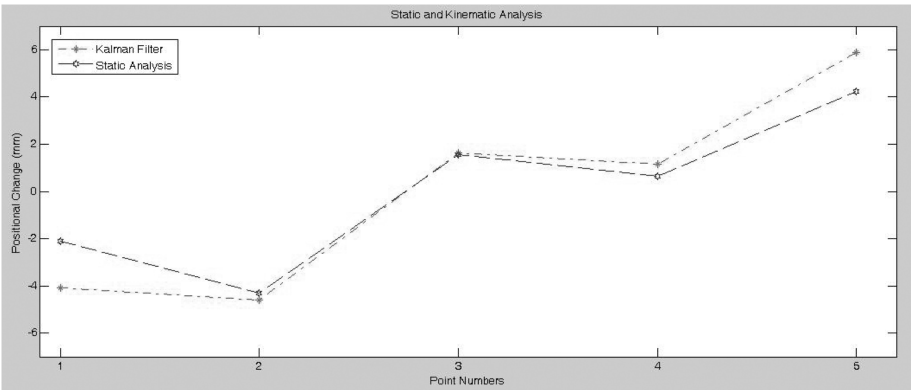

Figure 2. Displacement values determined with static and kinematic model

CONCLUSIONS

In this study, kinematic deformation analysis using Kalman filtering technique and quasi-static deformation analysis using S-transformation technique have been applied. Identical results were obtained for two different approaches. However, the obtained parameters using kinematic model are more than the obtained parameters using quasi-static model. In quasi-static model only geometric changes are obtained. In kinematic model velocity and acceleration values of points are obtained in addition to geometric changes. The main advantage of Kalman filtering technique is that it requires less measurement period. Since in Kalman filtering technique, the measurements have possible errors in nature and the state vector at a previous time is incorrect, the kinematic behaviors should not be extended unlimitedly by extrapolation.

ACKNOWLEDGMENT

The research is supported by the Coordinatorship of Selçuk University's Scientific Research Projects (Project No: 10701215).

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